GAME THEORY SOCIETY

北京大学国家发展研究院 National School of Development

GAMES 2024
PEKING UNIVERSITY
AUGUST 19-22, BEIJING

Decentralized multilateral bargaining

Yuan Ju (University of York)
Juan Vidal-Puga (Universidade de Vigo)

CITMAGA

ECOSOT Economics, Society and Territory
Universidade Vigo

MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES
Financiado por la Unión Europea NextGenerationEU
Plan de Recuperación, Transformación y Resiliencia
AGENCIA ESTATAL DE INVESTIGACIÓN

Motivation and contribution

1. A decentralized mechanism of multilateral negotiation:
   a. Generalizing the alternating-offer bargaining to n-player coalitional environment
   b. Counteroffers, partial agreements, local unanimity, no one excluded
2. Implement the Shapley NTU value
3. A solution theory synthesizes the Nash solution and Shapley value

Outline

1. Introduction: Nash program and NTU games
2. Non-cooperative game and the Shapley NTU value
3. Conclusion

Section 1: Introduction

Nash program and NTU games

Game theory

Cooperative game theory	Non-cooperative game theory

Game theory

Cooperative game theory	Non-cooperative game theory

- Implementation
- Nash program: Survey by Serrano (2005, 2021)
- Non-cooperative approach

NTU games

Transferable utility (TU) games

- Partial agreements
- Transferable utility
- Shapley value (1953)

NTU games

Transferable utility (TU) games

- Partial agreements
- Transferable utility
- Shapley value (1953)

Bargaining problems

- Unanimity required
- Non-transferable utility
- Nash solution (1950)

NTU games

Non transferable utility (NTU) games

- Partial agreements
- Non-transferable utility
- Harsanyi value (1963), Shapley NTU value (1969), consistent value (1989, 1992)

Transferable utility (TU) games

- Partial agreements
- Transferable utility
- Shapley value (1953)

Bargaining problems

- Unanimity required
- Non-transferable utility
- Nash solution (1950)

The model

A **Non-Transferable Utility (NTU) game** is a pair (N, V) where:

- $N = \{1, 2, ..., n\}$ is a set of players
- $V: S \subseteq N \longrightarrow V(S) \subset \mathbb{R}^S$ correspondence satisfying:
    - $V(S)$ non-empty, closed, convex, comprehensive, and bounded-above.
    - Superadditivity: $V(S) \mathrm{x} V(T) \subset V(S \cup T)$ for all $S, T \subset N$, $S \cap T = \varnothing$.
    - $V(S)$ nonlevel: For each x in the frontier of $V(S)$, there exists a unique normalized vector λ orthogonal to $V(S)$ on x with all its coordinates positive.

A **rule** is a function Φ that assigns to each NTU game (N,V) a payoff allocation $\Phi(N,V) \in V(N)$.

Example

Pure exchange economy with three players.

Coffee beans and water are required to prepare coffee. Sugar is optional.

- Player 1 has coffee beans, and prefers coffee with sugar.
- Player 2 has water.
- Player 3 has sugar.

Example

Pure exchange economy with three players.

Coffee beans and water are required to prepare coffee. Sugar is optional.

- Player 1 has coffee beans, and prefers coffee with sugar.
- Player 2 has water.
- Player 3 has sugar.

$$V(\{i\}) = \{x \in \mathbb{R}^{\{i\}} : x_i \leq 0\}$$
$$V(\{1,2\}) = \{x \in \mathbb{R}^{\{1,2\}} : 2x_1 + x_2 \leq 1\}$$
$$V(\{1,3\}) = \{x \in \mathbb{R}^{\{1,3\}} : x_1, x_3 \leq 0\}$$
$$V(\{2,3\}) = \{x \in \mathbb{R}^{\{2,3\}} : x_2, x_3 \leq 0\}$$
$$V(N) = \{x \in \mathbb{R}^{N} : x_1 + x_2 + x_3 \leq 1\}$$

The model

A **Transferable Utility (TU) game** is a pair (N, v) where:

- $N = \{1, 2, ..., n\}$ is a set of players
- $v: S \subseteq N \longrightarrow v(S) \in \mathbb{R}$ correspondence satisfying $v(\varnothing) = 0$.

The model

A **Transferable Utility (TU) game** is a pair (N, v) where:

- $N = \{1, 2, ..., n\}$ is a set of players
- $v: S \subseteq N \longrightarrow v(S) \in \mathbb{R}$ correspondence satisfying $v(\varnothing) = 0$.

Shapley value for TU games: It can be obtained from many different approaches:

- Axiomatic
- Marginalistic
- Potential
- Dividends

The model

A **Transferable Utility (TU) game** is a pair (N, v) where:

- $N = \{1, 2, ..., n\}$ is a set of players
- $v: S \subseteq N \longrightarrow v(S) \in \mathbb{R}$ correspondence satisfying $v(\emptyset) = 0$.

Shapley value for TU games: It can be obtained from many different approaches:

- ~~Axiomatic~~ (too loose)
- Marginalistic
- ~~Potential~~ (too strict)
- Dividends

The model

A **Transferable Utility (TU) game** is a pair (N, v) where:

- $N = \{1, 2, ..., n\}$ is a set of players
- $v: S \subseteq N \longrightarrow v(S) \in \mathbb{R}$ correspondence satisfying $v(\varnothing) = 0$.

Shapley value for TU games:

$Sh_i(N,v) = \sum_{S \subset N: i \in S} d_v(S)/\lvert S \rvert$ where $d_v(S) \in \mathbb{R}$ are the Harsanyi dividends of v.	$Sh_i(N,v) = \sum_{\pi \in \Pi} m_i^{\pi}(v)/\lvert \Pi \rvert$ where $m^{\pi}(v) \in \mathbb{R}^N$ are the marginal contributions vectors of v under order π.

TU games

Any TU game is also an NTU game.

TU games

Any TU game is also an NTU game.

$v(\{i\})=0$
$v(\{1,2\}) = 6$
$v(\{1,3\}) = 6$
$v(\{2,3\}) = 0$
$v(N) = 6$

TU games

Any TU game is also an NTU game.

$v(\{i\})=0$
$v(\{1,2\}) = 6$
$v(\{1,3\}) = 6$
$v(\{2,3\}) = 0$
$v(N) = 6$

$Sh(N,v) = (4,1,1)$

TU games

Any TU game is also an NTU game.

$v(\{i\})=0$ $\quad$ $V(\{i\}) = \{x \in \mathbb{R}^{\{i\}}: x_i \leq 0\}$

$v(\{1,2\}) = 6$ $\quad$ $V(\{1,2\}) = \{x \in \mathbb{R}^{\{1,2\}} : x_1 + x_2 \leq 6\}$

$v(\{1,3\}) = 6$ $\quad$ $V(\{1,3\}) = \{x \in \mathbb{R}^{\{1,3\}} : x_1 + x_3 \leq 6\}$

$v(\{2,3\}) = 0$ $\quad$ $V(\{2,3\}) = \{x \in \mathbb{R}^{\{2,3\}} : x_2 + x_3 \leq 0\}$

$v(N) = 6$ $\quad$ $V(N) = \{x \in \mathbb{R}^N: x_1 + x_2 + x_3 \leq 6\}$

$Sh(N,v) = (4,1,1)$

TU games

Any TU game is also an NTU game.

$v(\{i\})=0$ $\quad V(\{i\}) = \{x \in \mathbb{R}^{\{i\}} : x_i \leq 0\}$

$v(\{1,2\}) = 6$ $\quad V(\{1,2\}) = \{x \in \mathbb{R}^{\{1,2\}} : x_1 + x_2 \leq 6\}$

$v(\{1,3\}) = 6$ $\quad V(\{1,3\}) = \{x \in \mathbb{R}^{\{1,3\}} : x_1 + x_3 \leq 6\}$

$v(\{2,3\}) = 0$ $\quad V(\{2,3\}) = \{x \in \mathbb{R}^{\{2,3\}} : x_2 + x_3 \leq 0\}$

$v(N) = 6$ $\quad V(N) = \{x \in \mathbb{R}^N : x_1 + x_2 + x_3 \leq 6\}$

$Sh(N,v) = (4,1,1)$ $\quad Sh(N,V) = (4,1,1)$

TU games

Any TU game is also an NTU game.

If the utility is interchangeable at a fixed rate, the game is still (essentially) TU:

$$
\begin{aligned}
&v(\{i\})=0 \\
&v(\{1,2\}) = 6 \\
&v(\{1,3\}) = 6 \\
&v(\{2,3\}) = 0 \\
&v(N) = 6
\end{aligned}
$$

$$
\begin{aligned}
&V(\{i\}) = \{x \in \mathbb{R}^{\{i\}} : x_i \leq 0\} \\
&V(\{1,2\}) = \{x \in \mathbb{R}^{\{1,2\}} : x_1 + x_2 \leq 6\} \\
&V(\{1,3\}) = \{x \in \mathbb{R}^{\{1,3\}} : x_1 + x_3 \leq 6\} \\
&V(\{2,3\}) = \{x \in \mathbb{R}^{\{2,3\}} : x_2 + x_3 \leq 0\} \\
&V(N) = \{x \in \mathbb{R}^{N} : x_1 + x_2 + x_3 \leq 6\}
\end{aligned}
$$

$$
\begin{aligned}
&V(\{i\}) = \{x \in \mathbb{R}^{\{i\}} : \lambda_i x_i \leq 0\} \\
&V(\{1,2\}) = \{x \in \mathbb{R}^{\{1,2\}} : \lambda_1 x_1 + \lambda_2 x_2 \leq 6\} \\
&V(\{1,3\}) = \{x \in \mathbb{R}^{\{1,3\}} : \lambda_1 x_1 + \lambda_3 x_3 \leq 6\} \\
&V(\{2,3\}) = \{x \in \mathbb{R}^{\{2,3\}} : \lambda_2 x_2 + \lambda_3 x_3 \leq 0\} \\
&V(N) = \{x \in \mathbb{R}^{N} : \lambda_1 x_1 + \lambda_2 x_2 + \lambda_2 x_3 \leq 6\}
\end{aligned}
$$

$Sh(N,v) = (4,1,1)$ $\qquad$ $Sh(N,V) = (4,1,1)$

TU games

Any TU game is also an NTU game.

If the utility is interchangeable at a fixed rate, the game is still (essentially) TU:

$v(\{i\})=0$	$V(\{i\}) = \{x \in \mathbb{R}^{\{i\}}: x_i \leq 0\}$	$V(\{i\}) = \{x \in \mathbb{R}^{\{i\}}: \lambda_i x_i \leq 0\}$
$v(\{1,2\}) = 6$	$V(\{1,2\}) = \{x \in \mathbb{R}^{\{1,2\}}: x_1 + x_2 \leq 6\}$	$V(\{1,2\}) = \{x \in \mathbb{R}^{\{1,2\}}: \lambda_1 x_1 + \lambda_2 x_2 \leq 6\}$
$v(\{1,3\}) = 6$	$V(\{1,3\}) = \{x \in \mathbb{R}^{\{1,3\}}: x_1 + x_3 \leq 6\}$	$V(\{1,3\}) = \{x \in \mathbb{R}^{\{1,3\}}: \lambda_1 x_1 + \lambda_3 x_3 \leq 6\}$
$v(\{2,3\}) = 0$	$V(\{2,3\}) = \{x \in \mathbb{R}^{\{2,3\}}: x_2 + x_3 \leq 0\}$	$V(\{2,3\}) = \{x \in \mathbb{R}^{\{2,3\}}: \lambda_2 x_2 + \lambda_3 x_3 \leq 0\}$
$v(N) = 6$	$V(N) = \{x \in \mathbb{R}^N: x_1 + x_2 + x_3 \leq 6\}$	$V(N) = \{x \in \mathbb{R}^N: \lambda_1 x_1 + \lambda_2 x_2 + \lambda_2 x_3 \leq 6\}$
$Sh(N,v) = (4,1,1)$	$Sh(N,V) = (4,1,1)$	$Sh(N,V) = (4/\lambda_1, 1/\lambda_2, 1/\lambda_3)$

Money as utility

1. We give players money with exchange rates given by some $\lambda \in \mathbf{\Delta}^N$.

Money as utility

1. We give players money with exchange rates given by some $\lambda \in \mathbf{\Delta}^N$.
2. With such money acting as (transferable) utility, we have a TU game (N, v^λ).

Money as utility

1. We give players money with exchange rates given by some $\lambda \in \mathbf{\Delta}^N$.
2. With such money acting as (transferable) utility, we have a TU game (N,v^λ).
3. We compute $Sh(N,v^\lambda)$ using with this λ either the Harsanyi procedure or the average of marginal contributions vectors.

Money as utility

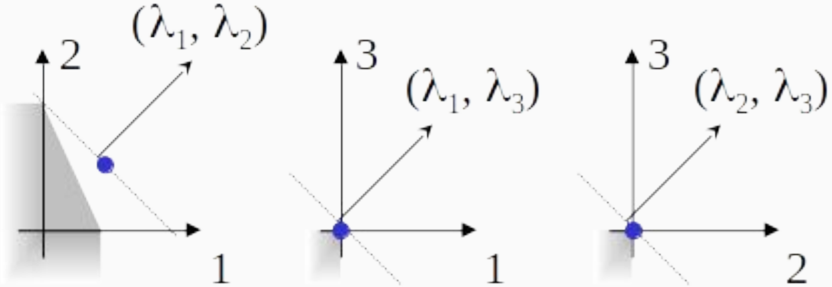
1. We give players money with exchange rates given by some $\lambda \in \mathbf{\Delta}^N$.
2. With such money acting as (transferable) utility, we have a TU game (N,v^λ).
3. We compute $Sh(N,v^\lambda)$ using with this λ either the Harsanyi procedure or the average of marginal contributions vectors.
4. If $Sh(N,v^\lambda) \in V(N)$, we say that $Sh(N,v^\lambda)$ is a **Shapley NTU** value of (N,V).

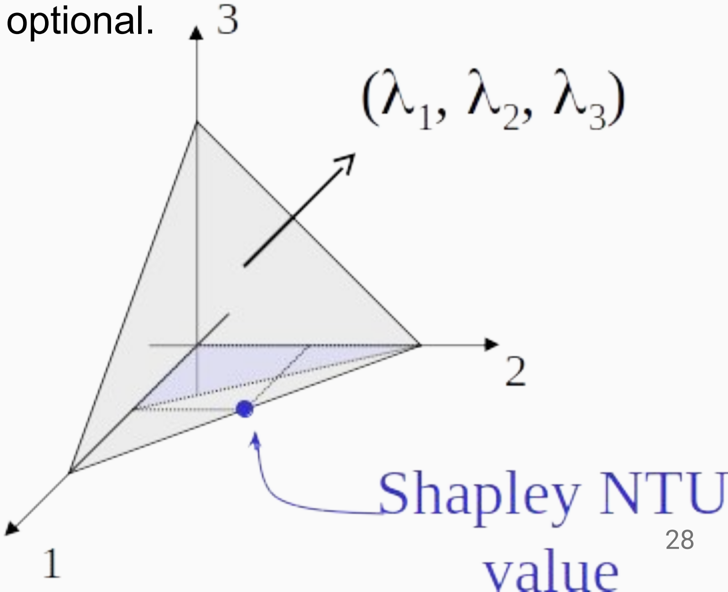
The Shapley NTU value (Shapley, 1969)

Pure exchange economy with three players.

Coffee beans and water are required to prepare coffee. Sugar is optional.

- Player 1 has coffee beans, and prefers coffee with sugar.
- Player 2 has water.
- Player 3 has sugar.





Money as utility (alternative 1)

1. We give players money with exchange rates given by $(\lambda^S)_{S \subseteq N}$ with $\lambda^S \in \Delta^S$ for all $S \subseteq N$.
   (Exchange rates depend on which players participate).

Money as utility (alternative 1)

1. We give players money with exchange rates given by $(\lambda^S)_{S \subseteq N}$ with $\lambda^S \in \Delta^S$ for all $S \subseteq N$.
   (Exchange rates depend on which players participate).
2. With such money acting as (transferable) utility in each coalition, we can use the Harsanyi procedure with λ^N in order to compute a payoff allocation $H(N, v^\lambda)$.

Money as utility (alternative 1)

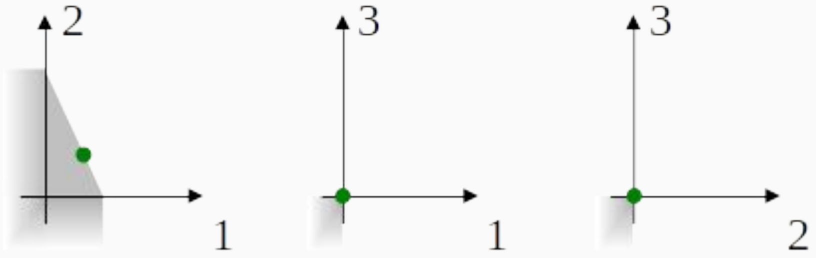
1. We give players money with exchange rates given by $(\lambda^S)_{S\subseteq N}$ with $\lambda^S \in \Delta^S$ for all $S\subseteq N$.
   (Exchange rates depend on which players participate).
2. With such money acting as (transferable) utility in each coalition, we can use the Harsanyi procedure with λ^N in order to compute a payoff allocation $H(N,v^\lambda)$.
3. If $H(N,v^\lambda) \in V(N)$, we say that $H(N,v^\lambda)$ is a **Harsanyi value** of (N,V).

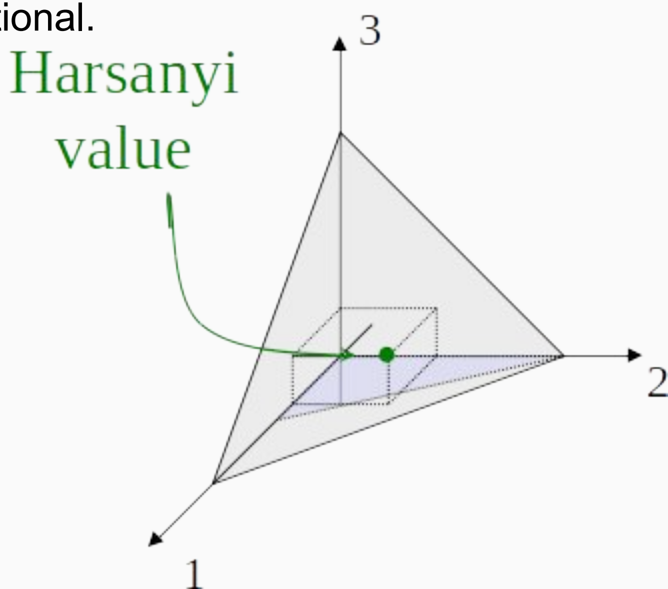
The Harsanyi value (Harsanyi, 1963)

Pure exchange economy with three players.

Coffee beans and water are required to prepare coffee. Sugar is optional.

- Player 1 has coffee beans, and prefers coffee with sugar.
- Player 2 has water.
- Player 3 has sugar.





Money as utility (alternative 2)

1. We give players money with exchange rates given by $(\lambda^S)_{S \subseteq N}$ with $\lambda^S \in \mathbf{\Delta}^S$ for all $S \subseteq N$.
   (Exchange rates depend on which players participate).

Money as utility (alternative 2)

1. We give players money with exchange rates given by $(\lambda^S)_{S\subseteq N}$ with $\lambda^S \in \mathbf{\Delta}^S$ for all $S \subseteq N$.
   (Exchange rates depend on which players participate).
2. With such money acting as (transferable) utility in each coalition, we can use the average of marginal contributions vectors with each λ^S in order to compute a payoff allocation $C(N,v^\lambda)$.

Money as utility (alternative 2)

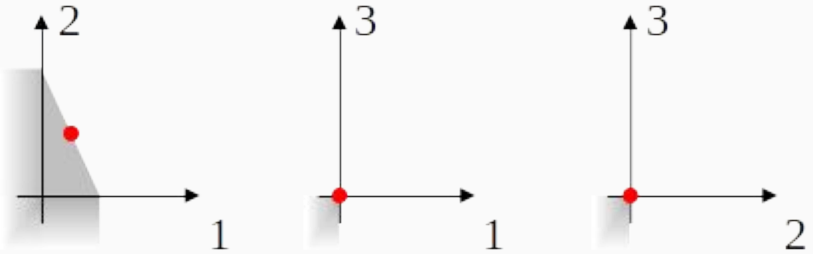
1. We give players money with exchange rates given by $(\lambda^S)_{S \subseteq N}$ with $\lambda^S \in \mathbf{\Delta}^S$ for all $S \subseteq N$.
   (Exchange rates depend on which players participate).
2. With such money acting as (transferable) utility in each coalition, we can use the average of marginal contributions vectors with each λ^S in order to compute a payoff allocation $C(N,v^\lambda)$.
3. If $C(N,v^\lambda) \in V(N)$, we say that $C(N,v^\lambda)$ is a **consistent value** of (N,V).

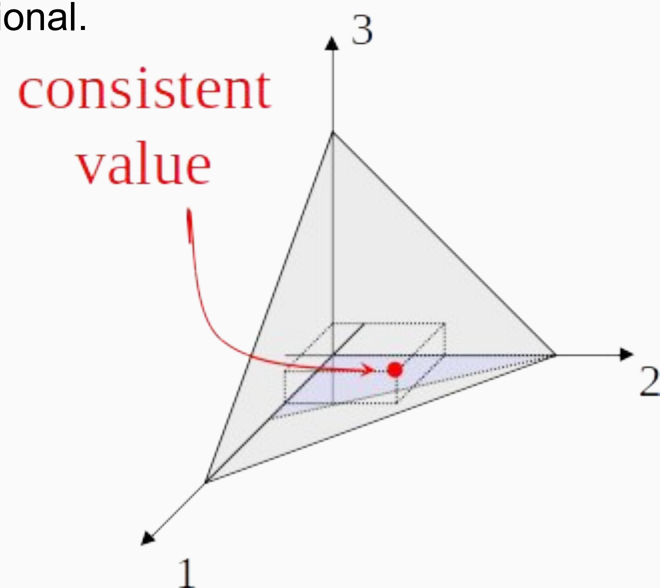
The consistent value (Maschler and Owen, 1992)

Pure exchange economy with three players.

Coffee beans and water are required to prepare coffee. Sugar is optional.

- Player 1 has coffee beans, and prefers coffee with sugar.
- Player 2 has water.
- Player 3 has sugar.





Generalizations of the Shapley value

			Exchange rate	
			Coalition dependent $(\lambda^S)_{S \subseteq N}$, $\lambda^S \in \mathbf{\Delta}^S \ \forall S \subseteq N$	Constant $\lambda \in \mathbf{\Delta}^N$
procedure	Harsanyi dividends	λ^S		
		λ^N		
	average of marginal contributions vectors			

Generalizations of the Shapley value

<table>
<tr><td colspan="3" rowspan="2"></td><td colspan="2">Exchange rate</td></tr>
<tr><td>Coalition dependent $(\lambda^S)_{S \subseteq N}$, $\lambda^S \in \mathbf{\Delta}^S \ \forall S \subseteq N$</td><td>Constant $\lambda \in \mathbf{\Delta}^N$</td></tr>
<tr><td rowspan="3">procedure</td><td rowspan="2">Harsanyi dividends</td><td>λ^S</td><td></td><td rowspan="3">Shapley NTU value</td></tr>
<tr><td>λ^N</td><td></td></tr>
<tr><td colspan="2">average of marginal contributions vectors</td><td></td></tr>
</table>

Generalizations of the Shapley value

<table>
<tr><td colspan="3" rowspan="2"></td><td colspan="2">Exchange rate</td></tr>
<tr><td>Coalition dependent $(\lambda^S)_{S \subseteq N}$, $\lambda^S \in \mathbf{\Delta}^S \ \forall S \subseteq N$</td><td>Constant $\lambda \in \mathbf{\Delta}^N$</td></tr>
<tr><td rowspan="3">procedure</td><td rowspan="2">Harsanyi dividends</td><td>λ^S</td><td></td><td rowspan="3">Shapley NTU value</td></tr>
<tr><td>λ^N</td><td>Harsanyi value</td></tr>
<tr><td colspan="2">average of marginal contributions vectors</td><td></td></tr>
</table>

Generalizations of the Shapley value

<table>
<tr><td colspan="3" rowspan="2"></td><td colspan="2">Exchange rate</td></tr>
<tr><td>Coalition dependent $(\lambda^S)_{S \subseteq N}$, $\lambda^S \in \mathbf{\Delta}^S \ \forall S \subseteq N$</td><td>Constant $\lambda \in \mathbf{\Delta}^N$</td></tr>
<tr><td rowspan="3">procedure</td><td rowspan="2">Harsanyi dividends</td><td>λ^S</td><td></td><td rowspan="3">Shapley NTU value</td></tr>
<tr><td>λ^N</td><td>Harsanyi value</td></tr>
<tr><td colspan="2">average of marginal contributions vectors</td><td>Consistent value</td></tr>
</table>

Generalizations of the Shapley value

<table>
<tr><td colspan="3" rowspan="2"></td><td colspan="2">Exchange rate</td></tr>
<tr><td>Coalition dependent $(\lambda^S)_{S\subseteq N}$, $\lambda^S \in \mathbf{\Delta}^S \ \forall S \subseteq N$</td><td>Constant $\lambda \in \mathbf{\Delta}^N$</td></tr>
<tr><td rowspan="3">procedure</td><td rowspan="2">Harsanyi dividends</td><td>λ^S</td><td>(Consistent Harsanyi value)</td><td rowspan="3">Shapley NTU value</td></tr>
<tr><td>λ^N</td><td>Harsanyi value</td></tr>
<tr><td colspan="2">average of marginal contributions vectors</td><td>Consistent value</td></tr>
</table>

Section 2

Non-cooperative game

Implementation of the Nash solution in bargaining games

- Nash (Econometrica, 1953)
- Rubinstein (Econometrica, 1982)
- van Damme (JET, 1986)
- Binmore (“The economics of bargaining”, ed. by Binmore and Dasgupta, 1987)
- Maschler, Owen and Peleg (“The Shapley value”, ed. by Roth, 1988)
- Hart and Mas-Colell (Econometrica, 1996)

Implementation of the Shapley value in TU games

- Gul (Econometrica, 1989)
- Hart and Moore (J Pol Ec, 1990)
- Winter (ET, 1994)
- Evans (GEB, 1992)
- Hart and Mas-Colell (Econometrica, 1996)
- Dasgupta and Chiu (IJGT, 1998)
- Pérez-Castrillo and Wettstein (JET, 2001)
- Vidal-Puga (EJOR, 2008)
- Ju (JME, 2012)

Common features when dealing with partial agreements

- Players "play" (*make offers and counteroffers, agree or disagree, vote, make partial payoffs, ...*) in N.
- Eventually, players split (or some are simply excluded) and the bargaining goes on in some (or several) subcoalition S, without possibility to rejoin.
- The risk of these splits is the tool that make players in N to reach an agreement in equilibrium.

Alternative features when dealing with partial agreements

- Players “play” (make offers and counteroffers, agree or disagree, vote, make partial payoffs, ...) in N, but their offers also consider the payoffs in case of disagreement.
- Players never split (nor are excluded) nor the bargaining goes on in some (or several) subcoalition S.
- The risk of disagreement is the tool that make players in N to reach an agreement in equilibrium.

Common and alternative features when dealing with partial agreements

- Players "play" (make offers and counteroffers, agree or disagree, vote, make partial payoffs, etc) in N.
- Eventually, players split (or some are simply excluded) and the bargaining goes on in some (or several) subcoalition S, without possibility to rejoin.
- The risk of these splits is the tool that make players in N to reach an agreement in equilibrium.

- Players "play" (make offers and counteroffers, agree or disagree, vote, make partial payoffs, etc) in N, but their offers also consider the payoffs in case of disagreement.
- **Players never split (nor are excluded) nor the bargaining goes on in some (or several) subcoalition S.**
- The risk of disagreement is the tool that make players in N to reach an agreement in equilibrium.

The non-cooperative game: Rounds 1 and 2

An order of the players is randomly chosen (assume $12...n$).

1. Player 1 presents a rule $f^{\{1\}}: S \subseteq N \longrightarrow f^{\{1\}}(S) \in V(S)$.
2. Player 2 either
   a. agrees on $f^{\{1\}}$ and joins $\{1\}$ (so coalition $\{1,2\}$ is formed), or
   b. disagrees and proposes a new rule $f^{\{2\}}$ to player 1.
      i. If player 1 accepts, $\{1,2\}$ forms with rule $f^{\{2\}}$, and the turn passes to player 3.
      ii. If player 1 rejects, the two player set apart for now, and the turn passes to player 3.

The non-cooperative game: Round r

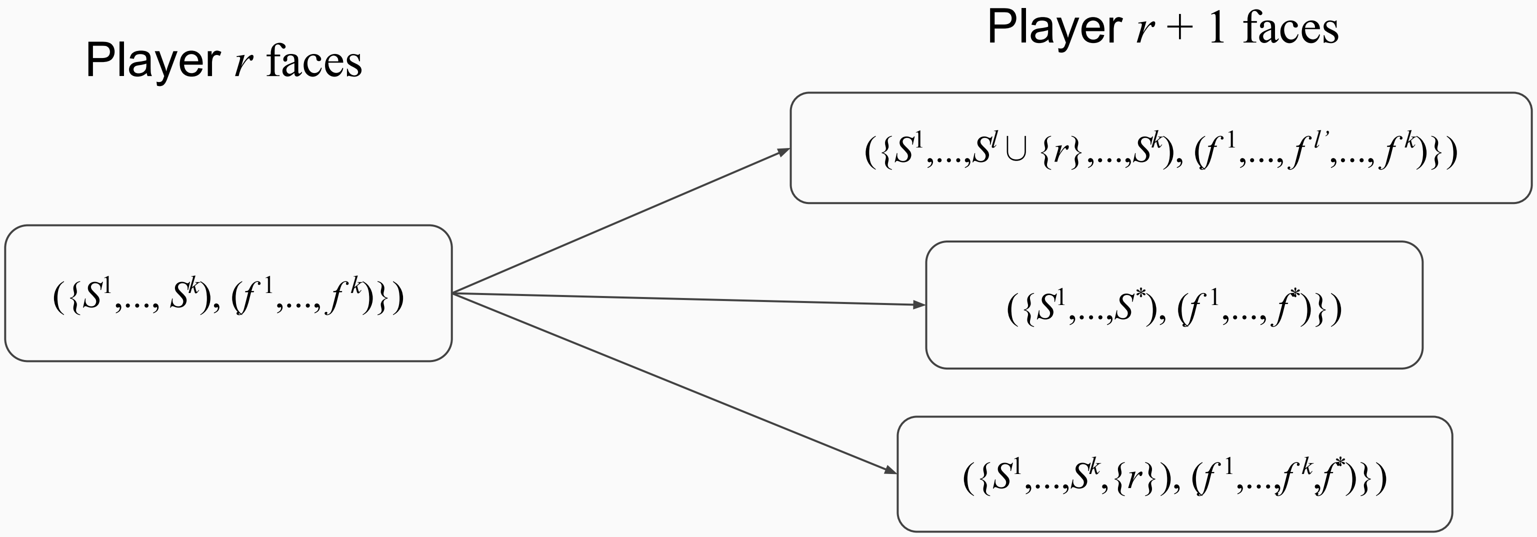
Player r faces $((S^1, f^1),...,(S^k, f^k))$ where

- $\{S^1,...,S^k\}$ is a partition of $\{1,...,r\text{-}1\}$ and
- $(f^1,...,f^k)$ is the vector of rules they have respectively agreed upon.

Player r either

1. agrees on some (S^l, f^l) and joins S^l, or
2. disagrees and proposes a new rule f^* to everyone.
   a. If some coalitions accept (unanimity required inside), they form a new merged coalition with r and rule f^*, and the turn passes to player $r + 1$.
   b. If all coalitions reject, player r does not join any coalition and the turn passes to $r + 1$ with $((S^1, f^1),...,(S^k, f^k),(\{r\}, f^*))$.

Round r


Player r faces
Player $r + 1$ faces
$(\{S^1,\ldots, S^k), (f^1,\ldots,f^k)\})$
$(\{S^1,\ldots,S^l \cup \{r\},\ldots,S^k), (f^1,\ldots,f^{l'},\ldots,f^k)\})$
$(\{S^1,\ldots,S^*), (f^1,\ldots,f^*)\})$
$(\{S^1,\ldots,S^k,\{r\}), (f^1,\ldots,f^k,f^*)\})$

Last round ($n + 1$)

- If we face $((\{N\}),(f))$, i.e., all coalitions have unanimously agreed on a single rule f, then each $i \in N$ receives $f_i(N)$ and the game finishes.
- If we face $((S^1,f^1),\ldots,(S^k,f^k))$ with $k > 1$, i.e., there is no unanimity, then
  - With $\varrho \in [0,1)$, the whole process is repeated with a (new) random order.
  - With $1 - \varrho$, each $i \in S^l$ receives $f_i^l(S^l)$ and the game ends.

Main result

There exists a stationary subgame perfect equilibrium payoff allocation for each order. Moreover, this payoff allocation is efficient and individually rational.

Furthermore, as ϱ approaches 1, the expected final payoff allocation approaches a Shapley NTU value.

Corollary:

- For TU games, the Shapley value is the unique expected equilibrium payoff.
- For bargaining problems, the unique expected equilibrium payoff approaches the Nash bargaining solution as ϱ approaches 1.

Section 3

Conclusion

Summary

Summary:

1. We design a decentralized protocol of bargaining (non-cooperative game) where no players are ever excluded.

2. We determine the final payoffs in equilibrium.

3. The final payoffs approach the Shapley NTU value.

Non-cooperative approaches

- Consistent value: Hart and Mas-Colell (Econometrica, 1996)
- Shapley NTU value: This research.
- Harsanyi value: Open question.