Homo Oeconomicus 29(2): 185–195 • (2012)

www.accedoverlag.de

A Violation of Monotonicity in a Noncooperative Setting

Maria Montero
School of Economics, University of Nottingham, UK
(eMail: maria.montero@nottingham.ac.uk)

Juan J. Vidal-Puga
Facultade de Ciencias Económicas e Empresariais, Universidade de Vigo, Spain
(eMail: vidalpuga@uvigo.es)

Abstract A power measure is monotone if a player with a larger weight is assigned at least as much power as a player with a smaller weight in the same weighted majority game. Failure of a power index to satisfy monotonicity is often considered a pathological feature. In this paper, we show that monotonicity may fail in the unique subgame perfect equilibrium of a noncooperative bargaining game. A player with a smaller weight may have a higher expected payoff than a player with a larger weight. This is possible even though coalition formation and payoff division are endogenous, all players are rational and there is no asymmetry between the players other than in the weights.

Keywords non-cooperative bargaining, power, violation of monotonicity

1. Introduction

A power measure has the monotonicity property if a larger player is always assigned at least as much power as a smaller player in the same weighted voting game. It is well known that some power indices like the public good index (Holler and Packel, 1983) and the Deegan-Packel (1978) index fail to satisfy this property. Violations of monotonicity have been considered unacceptable in a power measure (for example, Felsenthal and Machover, 1998, regard nonmonotonicity as a 'serious pathology').

ISBN 978-3-89265-080-5 ISSN 0943-0180

Holler and Napel (2004) question whether nonmonotonicity should automatically disqualify an index. They point out that there is more to a decision rule than weights and quota. Monotonicity may be violated due to the relative position of the players in a policy space. Owen (1971) and Shapley (1977) constructed generalizations of the Shapley-Shubik index assuming that players have an ideal point in the policy space and prefer policies that are closer to their ideal point; these power indices are not necessarily monotonic. Similarly, Laruelle and Valenciano (2005) note that a voter with a greater weight may be less likely to be decisive because of the probability distribution over vote configurations. If there is one large right-wing party and three small left-wing parties, it may be that the small parties tend to vote together in which case the large party cannot affect the outcome.

In this paper we provide an example of nonmonotonicity that does not rely on any asymmetry of players' preferences. There is a resource to be divided and players have symmetric preferences: each player would like to keep the whole resource. Players bargain over the resource according to a noncooperative bargaining procedure that treats all players symmetrically ex ante. Hence, the only way in which players differ from each other is in their weight. We will show that a player with a greater weight may have a lower expected payoff than a player with a smaller weight in the unique subgame perfect equilibrium of this game.

The noncooperative game is essentially the demand commitment bargaining model introduced by Morelli (1999) and further analyzed by Montero and Vidal-Puga (2007). In this game, players demand a share of the resource sequentially, and a coalition may form as soon as it has enough votes and its members have compatible demands. In case there is more than one feasible coalition, the player who made the last demand also decides which coalition forms. The order of moves is determined in advance by the first mover.

The weighted majority game in our example is an apex game. Apex games are weighted majority games with one major player (the apex player) and $n-1$ minor players (in order for the game to be asymmetric, n must be at least 4). There are two types of minimal winning coalitions: the apex player together with one of the minor players, and all the minor players together.

We will show that the apex player's expected payoff is smaller than that of a minor player, thus the equilibrium of this game violates monotonicity. By choosing a particularly favorable order of moves, a minor player is able to get the entire resource as a first mover; the apex player is not able to do the same. If all players have the same probability of being first mover, it follows that a minor player expects a greater share of the resource on average.

The intuition for this result is as follows. Suppose there are three minor players. If one of the minor players is chosen to be the first mover, it can choose an order of moves such that the apex player moves last and then demand the entire resource. The next minor player to move might then try to form a coalition with the apex player by demanding a positive payoff. However, this demand can always be undercut by the third minor player, so the second minor player is helpless: any attempts to form a coalition with the apex player will be sabotaged. Hence the second mover may as well go along with the first mover and demand 0. Once the second mover demands 0, the third mover is helpless as well: any positive demand on its part will result in the apex player forming a coalition with the second mover, so the third mover may as well go along with the first mover and demand 0. This is the unique subgame perfect equilibrium outcome of the game as we will discuss in section 2. In comparison, the apex player cannot demand more than $2/3$ as a first mover. If it demands more, there is no obstacle to the three minor players demanding $1/3$ each and excluding the apex player.[1]

2. The Model

2.1 *Simple Games*

A *simple game* is a pair (N,W) where $N = \{1,2,...,n\}$ be the set of players, and $W \subseteq 2^N$ be the list of winning coalitions. It is assumed that $N \in W$ and $\varnothing \notin W$. It is also assumed that extra players can never turn a winning coalition into a losing one ($T \in W$ whenever $S \in W$ for $S \subseteq T \subseteq N$) and two disjoint coalitions cannot both be winning ($S \in W$ and $T \in W$ implies $S \cap T \neq \varnothing$). We assume that there is a resource of size 1 to be divided, and W is the list of coalitions that are able to enforce a division of the resource.

The simple game is a *weighted majority game* if there is a vector $w = (w_1,...,w_n)$ of non-negative voting weights and a quota q such that S is winning iff $\sum_{i\in S} w_i \geq q$. Our assumptions above imply $0 < \frac{\sum_{i\in N} w_i}{2} < q \leq \sum_{i\in N} w_i$.

Apex games have one major player (the apex player) and $n-1$ identical minor players ($n \geq 3$). There are two types of minimal winning coalitions: the apex player together with one of the minor players, and all the minor players together. Apex games are a particular type of weighted majority games with $w = (n-2,1,...,1)$ and $q = n-1$.

[1] Montero and Vidal-Puga (2007) already noted that a player may be able to get the entire resource using the weighted majority game with $w = (3,2,2,1,1)$ and $q = 5$ as an example. Their example does not violate monotonicity because *all* players are able to obtain the entire resource by moving first, so that expected payoffs are $1/5$ for each player.

A *power measure* y is a mapping that assigns a non-negative real value to each player i in the simple game. We will denote the power of player i in the game (N,W) as $y_i(N,W)$, or simply y_i. A power measure is *monotone* if $w_i \geq w_j$ implies $y_i \geq y_j$ for any two players in the same weighted majority game $[q,w]$.

2.2 *The Noncooperative Bargaining Model*

Bargaining proceeds as follows. A player is chosen to be the first mover (we will assume each player is equally likely to be selected regardless of its weight). The first mover then chooses an order of moves for the remaining players. Once the order of moves is determined, each player i makes a payoff demand d_i, following the order of moves, where $d_i \in [0,1]$ is the share of the resource player i claims. If, after player i makes its demand, there exists a winning coalition S such that all members of S have already stated their demands and $\sum_{j \in S} d_j \leq 1$, player i has the additional choice of forming coalition S, in which case payoffs are distributed according to the demands made. If there is more than one possible S, player i decides which one is formed.[2] If the last mover forms no coalition, the game ends with each player getting zero.

2.3 *Example of Nonmonotonicity*

Consider the simplest possible asymmetric apex game with $w = (2,1,1,1)$ and $q = 3$. We now show that a minor player can obtain the whole resource when moving first, even though the rules of the bargaining procedure allow the next movers to form a coalition without the first mover.

Claim 1 Suppose a minor player is designated as the first mover, and chooses an order of moves so that the apex player moves last. Then the first mover obtains the entire resource in any subgame perfect equilibrium (SPE) of the game.

Proof Suppose a minor player (say, player 2) is designated as the first mover and chooses an order of moves so that the apex player moves last, such as 2, 3, 4, 1. We solve the game by backward induction. The challenge will be in showing the uniqueness of SPE payoffs, and for this we will need

[2] Note that player i is *not* forced to form a coalition whenever feasible; this feature of the game is essential to ensure the existence of a SPE at all subgames.

to show that players 3 and 4 must demand 0 in equilibrium after player 2 demands 1.

We start by analyzing the subgame in which player 1 gets the move. If player 1 gets the move, it faces demands d_2, d_3, d_4. If $1-\min(d_2,d_3,d_4) > 0$, it is optimal for player 1 to set $d_1 = 1-\min(d_2,d_3,d_4)$ and form a coalition with the minor player whose demand is minimal (or any of them if there is more than one). If $1-\min(d_2,d_3,d_4) = 0$, any $d_1 \in [0,1]$ is optimal for player 1, since no coalition can ever be formed unless $d_1 = 0$. The case $1-\min(d_2,d_3,d_4) < 0$ is not possible because $d_2,d_3,d_4 \leq 1$. Hence, there exists a SPE of this subgame and:

> If $1-\min(d_2,d_3,d_4) > 0$, then for each SPE of this subgame there exists some $i \in \arg\min_{j\in\{2,3,4\}} d_j$, such that player 1 sets $d_1 = 1-\min d_i$ and forms coalition $\{1,i\}$ (or a larger coalition if $d_j = 0$ for more than one $j \in \{2,3,4\}$).
>
> If $1-\min(d_2,d_3,d_4) = 0$, then there are multiple SPE of this subgame. In particular, for each $i \in \arg\min_{j\in\{2,3,4\}} d_j$, there exists a SPE such that the apex player sets $d_1 = 0$ and forms coalition $\{1,i\}$.

Not all SPE of this subgame can be extended to an SPE of the overall game because ties cannot always be broken in an arbitrary way (see Bennett and van Damme, 1991). Players may have to solve ties in a certain way in order to ensure that players moving earlier in the game have a best response as we will see below.

We now analyze the subgame in which player 4 gets the move. If player 4 gets the move, it faces demands d_2,d_3. Excluding dominated strategies, player 4 has two alternatives: setting $d_4 = 1-d_2-d_3$ (this is feasible if $1-d_2-d_3 \geq 0$) and forming coalition $\{2,3,4\}$, or setting some $d_4 \geq 1-d_2-d_3$ and let the apex player move. From the discussion above, we know that the apex player always forms coalition $\{1,4\}$ when $d_4 < \min(d_2,d_3)$ and never does it when $d_4 > \min(d_2,d_3)$. Moreover, the apex player can form a coalition with player 4 when $d_4 = \min(d_2,d_3)$. There are several possible cases, depending on how $1-d_2-d_3$ compares with $\min(d_2,d_3)$ and on whether player 4 can get a strictly positive payoff:

> If $1-d_2-d_3 < \min(d_2,d_3)$ and $\min(d_2,d_3) > 0$, then there exists a unique SPE. Player 4 sets $d_4 = \min(d_2,d_3)$, and the apex player sets $d_1 = 1-d_4$ and forms coalition $\{1,4\}$. The apex player must choose $\{1,4\}$ even though it is indifferent, because otherwise player 4 would be

optimizing in the open set $0 < d_4 < \min(d_2, d_3)$ and would not have a best response.

If $1 - d_2 - d_3 > \min(d_2, d_3)$ and $1 - d_2 - d_3 > 0$, then there exists a unique SPE. Player 4 sets $d_4 = 1 - d_2 - d_3$ and forms coalition $\{2,3,4\}$.

If $1 - d_2 - d_3 = \min(d_2, d_3) > 0$, there are two SPE in pure strategies: either player 4 forms $\{2,3,4\}$, or it gives the move to the apex player, who must then form coalition $\{1,4\}$ (the second of these two equilibria will not be extensible to the overall game as we will see below).

If $max(1 - d_2 - d_3, \min(d_2, d_3)) = 0$, then $\{d_2, d_3\} = \{0, 1\}$ (it follows from $d_2, d_3 \in [0,1]$) and hence $1 - d_2 - d_3 = \min(d_2, d_3) = 0$. There are multiple SPE in this case. In particular, there exists a SPE where player 4 sets $d_4 = 0$ and forms coalition $\{2,3,4\}$.

We now analyze the subgame in which player 3 gets the move. If player 3 gets the move, it faces demand d_2. From the above discussion, we know that player 3 can only get a positive payoff if $d_2 < 1$ and coalition $\{2,3,4\}$ is formed (any attempts to form a coalition with the apex player will be sabotaged by player 4). Coalition $\{2,3,4\}$ can only be formed if $1 - d_2 - d_3 \geq \min(d_2, d_3)$. We now look for the maximum value of d_3 that satisfies $1 - d_2 - d_3 \geq \min(d_2, d_3)$. Let f be the real-valued function $f(x) = 1 - d_2 - x$ and let g be the real-valued function $g(x) = \min(d_2, x)$. Since both functions are continuous, f strictly decreasing and g increasing, it is straightforward to check that the maximum value of x that yields $f(x) \geq g(x)$ is uniquely given by $x^* = 1 - 2d_2$ when $d_2 < \frac{1}{3}$, and $x^* = \frac{1-d_2}{2}$ when $d_2 > \frac{1}{3}$. In both cases, $x^* \in [0,1]$ and hence $d_3 = x^*$ is a feasible demand for player 3. When $x^* > 0$, any $d_3 \in (0, x^*)$ would induce $\{2,3,4\}$. Moreover, $x^* = 0$ iff $d_2 = 1$. Hence, there exists a SPE of this subgame and:

If $0 \leq d_2 \leq \frac{1}{3}$, then the SPE payoff is unique: player 3 sets $d_3 = 1 - 2d_2$, and player 4 sets $d_4 = 1 - d_2 - d_3$ and forms coalition $\{2,3,4\}$. Player 4 must form coalition $\{2,3,4\}$ because otherwise player 3 would be optimizing in the open set $d_3 < 1 - 2d_2$ and would not have a best response.

If $\frac{1}{3} < d_2 < 1$, then the SPE payoff is unique: player 3 sets $d_3 = \frac{1-d_2}{2}$, player 4 sets $d_4 = 1 - d_2 - d_3$ and coalition $\{2,3,4\}$ is formed. Player 4 must set $d_4 = 1 - d_2 - d_3$ because otherwise player 3 would be optimizing in the open set $d_3 < \frac{1-d_2}{2}$ and would not have a best response.

If $d_2 = 1$, then any $d_3 \in [0,1]$ is optimal for player 3, and there are multiple SPE payoffs. In particular, there exists a SPE in which player 3 sets $d_3 = 0$, player 4 sets $d_4 = 0$ and coalition $\{2,3,4\}$ is formed.

We now analyze the main game in which player 2 gets the (first) move. We have shown that any $d_2 < 1$ would induce coalition $\{2,3,4\}$, and $d_2 = 1$ may also induce coalition $\{2,3,4\}$. Hence, there exists a unique SPE where player 2 sets $d_2 = 1$, player 3 sets $d_3 = 0$, player 4 sets $d_4 = 0$ and coalition $\{2,3,4\}$ is formed. Player 3 and player 4 must set $d_3 = d_4 = 0$ because otherwise player 2 would be optimizing in the open set $d_2 < 1$ and would not have a best response. □

Minor players are able to exploit the asymmetry between the remaining players to their advantage. Given the order chosen by the first mover, there is no minimal winning coalition of players moving consecutively and immediately after the first mover, hence the remaining players are not able to coordinate on a reaction to an excessive demand on the part of the first mover. The apex player is not able to do the same because the remaining players are symmetric.

The case in which the apex player moves first is essentially a particular case of corollary 1 of Bennett and van Damme (1991). Their model is slightly different because each mover is chosen by the previous one rather than being determined in advance. However, since all minor players are identical, the order in which they move makes no difference to the apex player. The result is also a particular case of theorem 1 of Montero and Vidal-Puga (2011).

Claim 2 Suppose the apex player is designated as the first mover. Then the apex player demands $d_1 = \frac{2}{3}$ in any subgame perfect equilibrium.

Proof Suppose without loss of generality that the apex player chooses 1, 2, 3, 4 as the order of moves. The game can be solved by backward induction.

Player 4 compares $1 - d_1$ and $1 - d_2 - d_3$. It sets $d_4 = 1 - d_2 - d_3$ and forms the minor player coalition $\{2,3,4\}$ if $1 - d_2 - d_3 \geq 1 - d_1$; otherwise it sets $d_4 = 1 - d_1$ and forms a coalition with the apex player. Player 4 breaks ties in favor of the minor player coalition in order for players who move earlier to have a best response.

Player 3 has two options: it can either set $d_3 = 1 - d_1$ and form a coalition with the apex player, or it can set a demand that will induce player 4 to form the minor player coalition. Since player 4 solves ties in favor of the minor player coalition, the highest value of d_3 that still induces the minor player coalition is well defined as the solution of the equation $1-d_2-d_3 = 1-d_1$, which is $d_3 = d_1 - d_2$. Player 3 then chooses to induce the minor player coalition if $d_1 - d_2 \geq 1 - d_1$, or equivalently if $d_2 \leq 2d_1 - 1$. Ties must be solved in favor of the minor player coalition in order for player 2 to have a best response earlier on.

Player 2 has two options: it can either set $d_2 = 1-d_1$ and form a coalition with the apex player, or it can set $d_2 = 2d_1 - 1$ and induce the minor player coalition. Note that $d_2 = 2d_1 - 1$ can be written as $d_2 = 1-2(1-d_1)$: player 2 allows a payoff of $1-d_1$ for the other two minor players (so that they do not prefer to form a coalition with the apex player) and claims the residual.

Player 2 forms a coalition with the apex player iff $1-d_1 \geq 2d_1 - 1$, or equivalently if $d_1 \leq \frac{2}{3}$.

The apex player realizes that, in order to get a positive payoff, the highest demand it can make is $d_1 = \frac{2}{3}$, and this is the demand it makes in any SPE. (Note that if player 2 solved ties in favor of the minor player coalition the maximum achievable value of d_1 would not be well defined, which is why in order for strategies to constitute a SPE player 2 must solve ties in favor of player 1). □

There are two ways in which monotonicity is violated in the SPE of this game. If we look at what a player can get *conditional on being the first mover*, a small player is able to get more than the large player. If we look at the situation from an *ex ante* point of view, assuming that all players have the same probability of being first mover, it is still true that each small player has a greater expected payoff than the apex player. The apex player can get only $\frac{2}{3}$, and only when it is the first mover. A minor player can get 1 as a first mover (and may get $\frac{1}{3}$ with some probability, depending on how the apex player chooses the second mover).

3. Concluding Remarks

The lack of monotonicity of power indices such as the Deegan-Packel (1978) index[3] can be attributed to the exogenous nature of coalition formation and payoff division in their model. If we keep payoff division as assumed by Deegan and Packel but allow for endogenous coalition formation, it would not be possible for larger players to be worse-off than smaller players.

[3] Our results cannot be directly compared with the public good index since we assume that the resource to be divided is rival in consumption.

For example, consider the weighted majority game $[7;4,3,3,1,1]$. There are four minimal winning coalitions in this game, $\{1,2\}$, $\{1,3\}$, $\{2,3,4\}$ and $\{2,3,5\}$. Deegan and Packel assume that minimal winning coalitions must divide the total payoff equally, and each coalition is equally likely to form. Under this assumption, player 1 expects a payoff of $\frac{1}{4}$, whereas player 2 expects $\frac{7}{24} > \frac{1}{4}$. However, if we endogenize coalition formation and assume that a player will not join a particular coalition if there is another coalition in which it can get a greater payoff, coalitions $\{2,3,4\}$ and $\{2,3,5\}$ would be ruled out. The coalition structure core under equal payoff division would be $(\frac{1}{2},\frac{1}{2},0,0,0)$ and $(\frac{1}{2},0,\frac{1}{2},0,0)$, corresponding to coalitions $\{1,2\}$ and $\{1,3\}$. A coalition like $\{2,3,4\}$ with payoff division $(0,\frac{1}{3},\frac{1}{3},\frac{1}{3},0)$ would be vulnerable to a deviation by $\{1,2\}$ or $\{1,3\}$. If we accept that only coalitions with the lowest number of players will form, and that each of those is equally likely to form, a player with a larger weight must get at least the same payoff as a player with a smaller weight.[4]

It is worth noting that Gamson's (1961) model of coalition formation also violates monotonicity. In this model, it is assumed that the coalition that forms *must* divide the payoff proportionally to the players' weights. Players prefer to have a greater share of the resource, thus the winning coalition with the smallest total weight is unanimously preferred by its members (we can think of this theory as selecting the coalition structure core of a game with exogenous payoff division, as in the discussion above). For example, if weights are $4,3,2$ and the quota is 5, the player with 4 votes is always excluded. Players in this example are actually symmetric in terms of W, but monotonicity can also be violated when the larger player is strictly more desirable than the smaller player. In the example $[6;5,2,2,2]$, which is equivalent to $[3;2,1,1,1]$, the largest player is always excluded and has a payoff of 0. Nonmonotonicity in Gamson's model can be attributed to the assumption of exogenous payoff division: for example, in the game $[5;4,3,2]$, it can be argued that endogenous payoff division would lead to a symmetric situation in which all three members have the same power.

We have shown that nonmonotonicity is not confined to power indices, but can occur in the equilibrium of a noncooperative bargaining game with endogenous coalition formation and payoff division, even though players are rational and differ only in their weight. The bargaining procedure can be

[4] Let m be the least number of players in a winning coalition, let $\mathbb{S}$ be the set of winning coalitions with m players, and let $\mathbb{S}_k$ be the set of winning coalitions with m players involving player k. Player k's expected payoff would be $\frac{1}{m}\frac{|\mathbb{S}_k|}{|\mathbb{S}|}$. Let i and j be such that $w_i < w_j$. It is clear that $|\mathbb{S}_i| \leq |\mathbb{S}_j|$. This follows immediately if $\mathbb{S}_i$ is a subset of $\mathbb{S}_j$. If not, it is easy to see that for each coalition in $\mathbb{S}_i \backslash \mathbb{S}_j$ there exists a coalition in $\mathbb{S}_j \backslash \mathbb{S}_i$. Let T be a coalition in $\mathbb{S}_i \backslash \mathbb{S}_j$. Coalition $T' = T \backslash \{i\} \cup \{j\}$ is also winning and has m members, hence $T' \in \mathbb{S}_j \backslash \mathbb{S}_i$.

extended to allow for a finite number of bargaining rounds without affecting equilibrium payoffs.[5]

The lack of monotonicity of payoffs in our example could be avoided by modifying the noncooperative bargaining procedure. Bennett and van Damme (1991) study a more elaborated version in which each mover selects the next mover, so that the order of moves is not known in advance. Vidal-Puga (2004) assumes that the order of moves is randomly determined in advance, and only the last mover can form a coalition. Fréchette et al. (2005) assume that the next mover is randomly determined after each move. All these models lead to monotone expected equilibrium payoffs in our example, though some of them need refinements of SPE in order to achieve a unique prediction. Also using refinements, it can be shown that monotonicity is always satisfied in the proposal-based legislative bargaining model of Baron and Ferejohn (see Montero, 2012).

Acknowledgements

We are grateful to three anonymous referees for their comments. Juan Vidal-Puga thanks financial support by the Spanish Ministerio de Ciencia e Innovación through grant ECO2011-23460, and Xunta de Galicia through grant 10PXIB362299PR.

References

Baron, D. and J. Ferejohn (1989). 'Bargaining in legislatures'. *American Political Science Review* 87, 34–47.

Bennett, E. and E. van Damme (1991). 'Demand commitment bargaining: The Case of Apex Games'. *Game equilibrium models III: Strategic bargaining*. Ed. by R. Selten. Berlin. Springer-Verlag, 118–140.

Deegan, J., and E. W. Packel (1978). 'A new index of power for simple n-person games', *International Journal of Game Theory* 7, 113–123.

Felsenthal, D.S. and M. Machover (1998).*The measurement of voting power: Theory and practice, problems and paradoxes*, Edward Elgar Publishing.

Fréchette, G., Kagel, J. and M. Morelli (2005). 'Behavioral identification in coalitional bargaining: An experimental analysis of demand bargaining and alternating offers'. *Econometrica* 73, 1893–1937.

---

[5] This is straightforward if one of the players is randomly selected at the beginning to the first mover in all rounds. We show in the appendix of our working paper (Montero and Vidal-Puga, 2012) that the result also holds if a first mover is randomly selected at the beginning of each round: agreement still occurs in the first round and a minor player is able to demand 1 as a first mover, whereas the apex player is only able to demand $\frac{2}{3}$ as a first mover.

Gamson, W. A. (1961). 'A theory of coalition formation'. *American Sociological Review* 26, 373–382.

Holler, M.J. and S. Napel (2004). 'Monotonicity of power and power measures'. *Theory and Decision* 56, 93–111.

Holler, M.J. and E.W. Packel (1983). 'Power, luck and the right index'. *Journal of Economics* 43, 21–29.

Laruelle, A. and F. Valenciano (2005). 'A critical reappraisal of some voting power paradoxes'. *Public Choice* 125, 17–41.

Montero, M. and J.J. Vidal-Puga (2007). 'Demand commitment in legislative bargaining'. *American Political Science Review* 101, 847–850.

Montero, M. and J.J. Vidal-Puga (2011). 'Demand bargaining and proportional payoffs in majority games'. *Games and Economic Behavior* 71, 395–408.

Montero, M. and J.J. Vidal Puga (2012). 'A Violation of Monotonicity in a Noncooperative Setting'. Available at SSRN: http://ssrn.com/abstract=2021291 or http://dx.doi.org/10.2139/ssrn.2021291.

Montero, M. (2012). 'Postulates and paradoxes of voting power in a noncooperative setting'. Typescript.

Morelli, M. (1999). 'Demand competition and policy compromise in legislative bargaining'. *American Political Science Review* 93, 809–820.

Owen, G. (1971). 'Political games'. *Naval Research Logistics Quarterly* 18, 345–355.

Shapley, L.S. (1977). 'A comparison of power indices and a non-symmetric generalization'. The RAND Paper Series P-5872.

Vidal-Puga, J.J. (2004). 'Bargaining with commitments'. *International Journal of Game Theory* 33, 129–144.