Bargaining with commitments*

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Abstract

We study a simple bargaining mechanism in which, given an order of players, the first $n-1$ players sequentially announce their reservation price. Once these prices are given, the last player may choose a coalition to cooperate with, and pay each member of this coalition his reservation price. The only expected final equilibrium payoff is a new solution concept, the "selective value", which can be defined by means of marginal contributions vectors of a reduced game. The selective value coincides with the Shapley value for convex games. Moreover, for 3-player games the vectors of marginal contributions determine the core when it is nonempty.

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1 Introduction

Assume we study a society in which individuals have mechanisms which allow them to make binding agreements at no cost. In particular, the individuals are able to make commitments which cannot be revoked or renegotiated.

Consider a non-cooperative game, or mechanism, in which the players make demands following a pre-specified order. These demands represent the reservation prices for their respective resources. Finally, the last player in the order chooses the resources he wants to buy and clears the market.

The above protocol generalizes the bargaining game illustrated by Schelling (1980; Appendix B) as follows: two players may divide $100 as soon as they agree on how to do it. The game terminates at "midnight", when the bell rings. If the two players have jointly claimed more than $100, they get nothing. If they have jointly claimed no more than $100, they get their respective claims. The presence of commitments is illustrated by a "turnstile that permits a player to leave but not to return; his current offer as he goes through the turnstile remains on the books until the bell rings" (Schelling 1980, p. 276).

Several possible extensions of this mechanism for more than two players are given under the generic name of *Demand Commitment Game*. They are discussed in Bennet and van Damme (1991), Selten (1992), Winter (1994), and Dasgupta and Chiu (1998). A common feature of these models is that if one or more players "go through the turnstile" demanding a feasible amount (i.e. whatever they can assure by themselves is not less than the sum of their commitments), they can form a coalition and leave the game. Thus, some players may leave the game before all the others have a chance to move. In our model, every player (but the last one) commits to a reservation price. Our mechanism improves on previous ones in two aspects. First, it is simpler. The second aspect refers to efficiency. In Winter's and Dasgupta and Chiu's, the Shapley value (Shapley, 1953) arises for convex games. But if the game is not convex the equilibrium payoff may be inefficient[1]. In our mechanism the equilibrium outcome is always *unique* and *efficient* in a nonrestrictive class of games.

We call this (unique) outcome the *selective value*. Like the Shapley value, the selective value can be defined as the average of marginal contributions of the players given an order. The standard intuition of the marginal contributions in the Shapley value is as follows: Players sequentially enter a room, and each player gets the difference between the worth of the people in the room before and after his arrival. Of course, an equivalent interpretation is that the players are initially cooperating in the room, and they sequentially leave getting their respective marginal contribution.

The intuition of the marginal contributions in the selective value is similar. All the players are initially cooperating in a room. Upon leaving the room, each player prices his resources as the difference between the worth of the people in the room before and after he leaves. However, at each point in time, the players left in the room continue to think that they are cooperating and may choose the "optimal" coalition consisting of players who have already left.

Thus, unlike with the Shapley value, these marginal contributions are not computed from the characteristic function of the original game. Instead, they come from the characteristic function of a reduced game (following the ideas in Davis and Maschler (1965) and Peleg (1986)) where a coalition can buy the resources of the other players.

[1] In Dasgupta and Chiu's model, efficiency in the nonconvex case is achieved by means of prizes and penalties from the planner to the players. For large enough penalties, the Shapley value arises for any game, and the planner does not gain or loose anything in equilibrium. I think this result is unsatisfactory. For example, out of the equilibrium path there may be a utility transfer to the players from outside.

Hence, the selective value relates the definition of the Shapley value as an average of marginal contributions to Davis and Maschler's ideas of reduced games[2]. Both kinds of ideas have been successfully, but separately, explored in the non-cooperative implementation of the Shapley value (Gul (1989), Winter (1994), Hart and Mas-Colell (1996), Krishma and Serrano (1995), Pérez-Castrillo and Wettstein (2001), among others), the core (Serrano (1995a)), the nucleolus (Serrano (1993, 1995b)) and the kernel (Serrano (1997)).

The selective value coincides with the Shapley value for convex games, and thus we extend results due to Winter (1994) and Dasgupta and Chiu (1998). For 3-player games, Dasgupta and Chiu show that the possible outcomes are the vertices of the core, when the core has nonempty interior. We prove that this result also applies in our mechanism. When the core is empty, the selective value coincides with the nucleolus. Furthermore, the selective value is also characterized for simple games.

In Section 2, we introduce our notation. In Section 3, we define the selective value and illustrate some of its properties. We also study the selective value in several important classes of games. In Section 4, we formally describe the non-cooperative mechanism[3] and prove that the selective value is the only expected final payoff in subgame perfect equilibrium. Since the selective value coincides with the Shapley value for convex games, we then have given an additional non-cooperative justification for the Shapley value for this class of games. In Section 5 we present some concluding remarks.

[2] I thank an anonymous referee for suggesting the relation between the selective value and the reduced game.

[3] To avoid ambiguities, we use the term *non-cooperative mechanism*, or simply *mechanism*, when referring to a non-cooperative game.

2 Notation

Given a finite set A, by 2^A we denote the cardinal set of A, by $|A|$ the cardinality of A, and by $\mathbb{R}^A$ the set of real $|A|$-tuples whose indices are the elements of A. Given a function $f : 2^A \to \mathbb{R}$, by $\arg\max_{T \subset S} \{f(T)\}$ we denote the set of subsets $T \subset S$ that maximize $f(T)$.

Let (N, v) be a transferable utility game (*TU game*), where $N = \{1, 2, ..., n\}$ is the set of *players* and v is the *characteristic function*. This function assigns a real number $v(S)$ to every *coalition* $S \subset N$, $S \neq \emptyset$ and $v(\emptyset) = 0$. The number $v(S)$ represents the utility that players in S are able to achieve by themselves when playing cooperatively. We often refer to "the game v" instead of "the TU game (N, v)". Let $TU(N)$ denote the class of all games with player set N and TU the class of all TU games.

Say that v is *convex* if $v(T) - v(T \setminus \{i\}) \leq v(S) - v(S \setminus \{i\})$ when $i \in T \subset S$, *zero-monotonic* if $v(S) + v(\{i\}) \leq v(S \cup \{i\})$ when $i \notin S$, and *strictly zero-monotonic* if $v(S) + v(\{i\}) < v(S \cup \{i\})$ when $i \notin S$. Note that if a game is convex, it is zero-monotonic. Say that v is *monotonic* if $v(T) \leq v(S)$ whenever $T \subset S$.

The *core* $C(v)$ of the game v is the set of vectors $x \in \mathbb{R}^N$ such that $\sum_{i \in N} x_i = v(N)$ and $\sum_{i \in S} x_i \geq v(S)$ for all $S \subset N$. The core of a game may be empty. However, if the game is convex, its core is nonempty.

Let Π be the set of all orders of N. Given $\pi \in \Pi$ and $i \in N$, we define the set of predecessors of i under π as

$$P_i^\pi := \{j \in N : \pi(j) < \pi(i)\}.$$

We also denote $\overline{P_i^\pi} := P_i^\pi \cup \{i\}$.

In contrast with the standard motivation given in the literature, we define the marginal contribution of a player in terms of people sequentially leaving a room, all players being initially present and cooperating.

Thus, the *marginal contribution* of player i in the game v under the order π is given by

$$m_i^{\pi}(v) := v\left(N \backslash P_i^{\pi}\right) - v\left(N \backslash \overline{P_i^{\pi}}\right). \quad (1)$$

The *Weber set* of v, $W(v)$, is the convex hull of the vectors $m^{\pi}(v)$'s. If v is convex, $W(v) = C(v)$.

A *value* on $G \subset TU(N)$ is a map $f : G \longrightarrow \mathbb{R}^N$. The *Shapley value* (Shapley, 1953) for the game v is the average of the vectors of marginal contributions, namely

$$Sh(v) := \frac{1}{|\Pi|} \sum_{\pi \in \Pi} m^{\pi}.$$

Other significant values are the *prenucleolus* and the *nucleolus* (Schmeidler, 1969). On the class of 3-player zero-monotonic games, the prenucleolus and the nucleolus coincide.

We say that a value f on G satisfies *efficiency* if for all $v \in G$, $\sum_{i \in N} f_i(v) = v(N)$. It satisfies *core selection* if $f(v) \in C(v)$ for all $v \in G$ such that $C(v) \neq \emptyset$. The Shapley value satisfies efficiency but not core selection. The prenucleolus and the nucleolus satisfy both efficiency and core selection.

We say that v is a *simple game* if $v(S) \in \{0, 1\}$ for all $S \subset N$, and $v(N) = 1$. A coalition S in a single game is *winning* if $v(S) = 1$. Player i in a simple game is a *veto player* if $i \notin S$ implies $v(S) = 0$. The core of a monotonic simple game v with set of veto players $T \subset N$ is the convex hull of the imputations $x \in \mathbb{R}^N$ which satisfy $x_i \geq 0$ for all $i \in T$, $\sum_{i \in T} x_i = 1$ and $x_i = 0$ for all $i \in N \backslash T$. Thus, the core of v is nonempty if and only if $T \neq \emptyset$.

3 The selective value

In this section we define the selective marginal contribution of a player for each order $\pi \in \Pi$. Like with the marginal contributions that determine the Shapley value, imagine the players sequentially leave a room. Again, a player's marginal contribution is the difference between what the grand coalition can get by itself before and after he leaves. However, in computing what a coalition can get by itself, we assume that it may *select* some of the players already outside of the room and have access to their resources, upon paying their reservation prices. This means that, when $P \subset N$ is the set of players outside of the room and $(r_i)_{i \in P} \in \mathbb{R}^P$ is the vector of their reservation prices, the game we are considering is not (N, v), but rather the *reduced game* $(N \backslash P, v_r)$ where

$$v_r(T) = \begin{cases} 0 & \text{if } T = \emptyset \\ \max_{S \subset P} \left\{ v(T \cup S) - \sum_{i \in S} r_i \right\} & \text{otherwise} \end{cases} \tag{2}$$

for all $T \subset N \backslash P$. Note that, since the maximum is unique, v_r is well-defined.

This game is similar to one proposed by Davis and Maschler (1965) and Peleg (1986). In the Davis-Maschler-Peleg (from now on, DMP) reduced game, $v_r(N \backslash P) = v(N) - \sum_{i \in P} r_i$, i.e. the players in P should receive their components of r. A consistency notion based on this definition of a reduced game has provided the basis for characterizations of the core and the prekernel (Peleg, 1986) and the prenucleolus (Sobolev, 1975).

Peleg (1986) explains the idea of the reduction as follows: Players in $N \backslash P$ have agreed that the members of P should get their components of r, and they have to decide how to share the surplus $v(N) - \sum_{i \in P} r_i$. Furthermore, the members of P, *subject to getting their components of* r, continue to cooperate with the members of $N \backslash P$. Then, for every coalition $T \subset N \backslash P$, $v_r(T)$ is the total payoff that the members of T expect to get.

In contrast, in the reduced game given by (2), players in $N \backslash P$ do not assume that the members of P should get their components of r. However, and remarkably, this distinction vanishes in our definition, as shown in Lemma 1 below.

Let $\pi \in \Pi$. We define the *selective marginal contribution* of player $i \in N$ under π as

$$s_i^\pi (v) := v_s (N \backslash P_i^\pi) - v_s \left(N \backslash \overline{P_i^\pi} \right) \tag{3}$$

where $s = \left(s_j^\pi (v) \right)_{j \in P_i^\pi} \in \mathbb{R}^{P_i^\pi}$. Note that the definition of $s_i^\pi (v)$ depends only on those components $s_j^\pi (v)$ that come before player i's in π; this is why $s^\pi (v)$ is well-defined and unique by induction.

Note that the only difference between the marginal contributions given in (1) and (3) is that what a coalition can get by itself is not given by v, but by v_s. When it is player i's turn to leave, he selects the coalition $N \backslash S$ for whom the surplus left to the coalition consisting of the remaining players is maximal. Thus, each member of group S priced himself out of the market. However, player i cannot claim the whole surplus, because then he would also become too expensive for the players leaving after him.

Note that for $\pi (1) = 1$, $P_1^\pi = \emptyset$. Thus, the reduced game coincides with v and the marginal contribution of player 1 is the same in both models, namely

$$s_1^\pi (v) = v (N) - v (N \backslash \{1\}) = m_1^\pi (v) . \tag{4}$$

The next lemma shows that the game v_s coincides with the DMP reduced game.

Lemma 1 *Given $i \in N$ and $\pi \in \Pi$,*

$$v_s (N \backslash P_i^\pi) = v (N) - \sum_{j \in P_i^\pi} s_j^\pi (v) ,$$

that is, no predecessor in the order is ever excluded by player i.

Proof. We assume $\pi = (12...n)$. We proceed by induction on i. For $i = 1$ the result is trivial, since $P_1^\pi = \emptyset$. Assume the result is true for $1, 2, ..., i-1$. We need to prove that, for any $S \subsetneq P_i^\pi$, the surplus obtained by including only the members of S is not less than the surplus obtained by including all P_i^π, i.e.

$$v\left(\left(N \backslash P_i^\pi\right) \cup S\right) - \sum_{j \in S} s_j^\pi(v) \leq v(N) - \sum_{j \in P_i^\pi} s_j^\pi(v). \tag{5}$$

Since $S \neq P_i^\pi$, there are a player $k < i$ and a coalition $T \subset P_k^\pi$ such that $S = T \cup \{k+1, k+2, ..., i-1\}$. By the induction hypothesis

$$s_k^\pi(v) = v(N) - \sum_{j \in P_k^\pi} s_j^\pi(v) - \max_{S' \subset P_k^\pi} \left\{ v\left(\left(N \backslash \overline{P_k^\pi}\right) \cup S'\right) - \sum_{j \in S'} s_j^\pi(v) \right\},$$

which, by taking $S' = T$, gives

$$\begin{aligned} s_k^\pi(v) &\leq v(N) - \sum_{j \in P_k^\pi} s_j^\pi(v) - v\left(\left(N \backslash \overline{P_k^\pi}\right) \cup T\right) + \sum_{j \in T} s_j^\pi(v) \\ &= v(N) - \sum_{j \in P_i^\pi \backslash \{k\}} s_j^\pi(v) - v\left(\left(N \backslash P_i^\pi\right) \cup S\right) + \sum_{j \in S} s_j^\pi(v), \end{aligned}$$

from which (5) is easily deduced. ■

Lemma 1 has two important consequences. First, for any $i \in N$, the game $(N \backslash P_i^\pi, v_s)$ defined as in (2) is a DMP reduced game for every P_i^π. This means that the selective marginal contribution of player i given in (3) is his marginal contribution in the DMP reduced game. Second, for each order, the aggregate payoff is exactly $v(N)$. This is not the case in the DMP reduced game, where the players in a coalition $N \backslash P$ assume (given a vector $r \in \mathbb{R}^N$) that players in P should get their components of r.

Remark 1 *While Lemma 1 shows that the first term in (3) never excludes any player, the second term* $v_s\left(N \backslash \overline{P_i^\pi}\right)$ *may exclude some players. Note that the selective marginal contributions of the predecessors of player* i *were computed* assuming that player i was cooperating. *Without player* i*, the demands of his predeccesors may become too expensive. For example, let* $i = 2$*, and* $\alpha := (12...n) \in \Pi$ *with* $n \geq 3$*, and let* v *be the unanimity game,* *i.e.* $v(N) = 1$ *and* $v(S) = 0$ *if* $S \neq N$. *The first player, when leaving, demands* $s_1^\alpha(v) = 1$. *Thus,*

$$v_s\left(N \backslash \overline{P_2^\alpha}\right) = 0 \neq -1 = v(N \backslash \{2\}) - s_1^\alpha(v)$$

i.e. the players in the room ($N \backslash \{1\}$*) can get an aggregate payoff of* 0 *counting on player* 1*, but without player* 2 *the cooperation of player* 1 *is unnecessary and thus there is no point in paying his demand.*

Lemma 1 also provides the following simple formula for $s^\pi(v)$:

$$s_i^\pi(v) = v(N) - \max_{T \subset P_i^\pi}\left\{v(N \backslash (T \cup i)) + \sum_{j \in T} s_j^\pi(v)\right\}$$

for $\pi \in \Pi$ and $i \in N$ such that $\pi(i) < n$.

The next lemma establishes that, whatever aggregate payoff the members of $N \backslash P$ may get in the reduced game (by cooperating with players in P), this amount is not less than what they get if one of them leaves and the remaining players in $N \backslash P$ cooperate with the players in P.

Lemma 2 *If* v *is zero-monotonic, then*

$$v_r(N \backslash P) \geq v_r(N \backslash (P \cup \{i\})) + v(\{i\})$$

for all $i \in N \backslash P$ *and all* $(r_j)_{j \in P} \in \mathbb{R}^P$. *If* v *is strictly zero-monotonic, the inequality is strict.*

Proof. Let $P \subset N$, $i \in N \backslash P$, and $(r_j)_{j \in P} \in \mathbb{R}^P$. Let $E \subset N$ be such that

$$E \in \arg\max_{T \subset P}\left\{v(N \backslash (T \cup \{i\})) - \sum_{j \in P \backslash T} r_j\right\}.$$

By zero-monotonicity,

$$v(N \backslash E) - \sum_{j \in P \backslash E} r_j \geq v(N \backslash (E \cup \{i\})) + v(\{i\}) - \sum_{j \in P \backslash E} r_j$$

which is precisely $v_r(N\backslash(P\cup\{i\}))+v(\{i\})$. Hence, the result holds. The proof for the strict inequality is analogous. ■

Remark 2 *The reduced game v_r need not satisfy this weak version of zero-monotonicity (i.e. $v_r(N\backslash P)\geq v_r(N\backslash(P\cup\{i\}))+v_r(\{i\})$) because both $N\backslash(P\cup\{i\})$ and $\{i\}$ may be using the same resources. See Example 2.7 in Peleg (1986).*

Corollary 1 *Lemma 2 implies that each $s^\pi(v)$ is individually rational for v (i.e. $s_i^\pi(v)\geq v(\{i\})$ for all $i\in N$) when v is zero-monotonic.*

Analogously to the Weber set, we define $W^\sigma(v)$ as the convex hull of the vectors $s^\pi(v)$'s.

Definition 1 *Given a game v, its* selective value $\sigma(v)$ *is the average of its selective marginal contributions vectors, namely*

$$\sigma(v):=\frac{1}{|\Pi|}\sum_{\pi\in\Pi}s^\pi(v).$$

The next proposition characterizes the selective value in convex games, monotonic simple games, and zero-monotonic 3-player games.

Proposition 1 *a) If v is convex, then $\sigma(v)=Sh(v)$.*

b) Let v be a monotonic simple game, and let T be the set of veto players. Then, $\sigma(v)$ is given by

1. *If $T=\emptyset$*

$$\sigma_i(v)=\frac{1}{n}$$

for all $i\in N$.

2. *If $T\neq\emptyset$*

$$\sigma_i(v)=\begin{cases}\frac{1}{|T|} & \text{if } i\in T\\ 0 & \text{if } i\notin T.\end{cases}$$

c) Let v be a zero-monotonic game with $n = 3$.

1. *If the core of v is nonempty, then the selective marginal contributions vectors of v are its vertices. Thus, $W^{\sigma}(v) = C(v)$.*

2. *If the core is empty, then the selective value coincides with the prenucleolus.*

The proof of Proposition 1 is in the Appendix. The proof of the next corollary is straightforward.

Corollary 2 *The selective value satisfies core selection for convex games, monotonic simple games, and zero-monotonic* 3*-player games.*

Proposition 1, part c1), together with the results in the next sections, also extend Theorem 5 in Dasgupta and Chiu (1998). Moreover, Proposition 1, part a) extends the results of Winter (1994) and Dasgupta and Chiu (1998) (cf. Theorem 1 below) for convex games. For non-convex games, the selective value does not coincide with the Shapley value (see Examples 1 and 2 below). However, it is more "stable" (in the sense of "core selection") in both simple games (Proposition 1, part b)) and 3-player games (Proposition 1, part c1)). Example 1 shows that this stability result does not hold for more than 3 players. Moreover, it shows that parts a) and c1) in Proposition 1 are tight.

Example 1 *Let $n = 4$ and v be given by $v(\{i\}) = 0$ for all $i \in N$, $v(N) = 60$, $v(S) = 36$ if $\{1, 2\} \subset S$ and $|S| \leq 3$, and $v(S) = 24$ otherwise. Then, $Sh(v) = (18, 18, 12, 12) \in C(v)$, and $\sigma(v) = (19, 19, 11, 11) \notin C(v)$.*

Example 2 *Let $n = 4$ and v be given by $v(\{i\}) = 0$ for all $i \in N$, $v(N) = 60$, $v(S) = 36$ if $1 \in S$ and $2 \leq |S| \leq 3$, and $v(S) = 24$ otherwise. Then, $Sh(v) = (21, 13, 13, 13) \notin C(v)$, and $\sigma(v) = (24, 12, 12, 12) \in C(v)$.*

Proposition 1, part c2), is also tight. The next example shows that the selective value for 3 players does not coincide with the prenucleolus when the game has a nonempty core.

Example 3 *Let* $n = 3$ *and* v *be given by* $v(N) = 30$*,* $v(\{1,2\}) = v(\{1,3\}) = 6$ *and* $v(S) = 0$ *otherwise. Then,* $\sigma(v) = (12, 9, 9)$ *and the prenucleolus of* v *is* $(10, 10, 10)$*.*

Now, we briefly discuss whom in the order has more power. We assume that the order is $\alpha = (1...n) \in \Pi$. If only the grand coalition is winning (unanimity games) then the first player gets all the surplus. On the other hand, if the game has no veto players, then the last player gets all the surplus. When there are veto players, each of them would prefer to be the first veto player in the order.

For a zero-monotonic game v, the first player gets his marginal contribution (as stated in (4)). When v is convex, $m^{\alpha}(v) = s^{\alpha}(v)$ (see the proof of Proposition 1, part a)). In a convex game, the larger the coalition, the larger the marginal contribution of a new member. Thus, players prefer to appear in the order as soon as possible. Moreover, the last player obtains his minimum possible payoff $v(\{n\})$.

The power of the last player, however, can significantly increase when the game is not convex. Consider for example the symmetric game v with $n = 3$ given by $v(N) = 1$, $v(S) = \nu$ if $|S| = 2$, and $v(S) = 0$ otherwise. The selective marginal vector is $s^{\alpha}(v) = (1 - \nu, \nu, 0)$ for $\nu \leq 1/2$ and $s^{\alpha}(v) = (1 - \nu, 1 - \nu, 2\nu - 1)$ for $\nu > 1/2$. Note that this game is convex for $\nu \leq 1/2$.

A reason for the increase in the last player's power is the following. Players in the middle are constrained by the first players' reservation prices. They cannot demand too much because they would become too expensive. If a player is forced to demand little, the players after him may need to reduce their own demands, so as not to be excluded. The next example illustrates this point: Consider the symmetric game v with $n = 4$ given by $v(N) = 1$, $v(S) = \nu$ if $|S| = 2$ or $|S| = 3$, and $v(S) = 0$ otherwise. The selective marginal vector is $s^{\alpha}(v) = (1 - \nu, 0, 0, \nu)$.

For the sake of completeness, we also consider the symmetric game v with $n = 4$ given by $v(N) = 1$, $v(S) = \nu_3$ if $|S| = 3$, $v(S) = \nu_2$ if $|S| = 2$, and $v(S) = 0$ otherwise. This game is convex for $2\nu_2 \leq \nu_3 \leq (\nu_2 + 1)/2$. Its selective marginal vector $s^{\alpha}(v)$ is

$$\begin{array}{ll} (1-\nu_3, \nu_3-\nu_2, \nu_3-\nu_2, 2\nu_2-\nu_3) & \text{for } \nu_3 \leq \min\left\{2\nu_2, \frac{\nu_2+1}{2}\right\}, \\ (1-\nu_3, \nu_3-\nu_2, \nu_2, 0) & \text{for } 2\nu_2 \leq \nu_3 \leq \frac{\nu_2+1}{2}, \\ (1-\nu_3, 1-\nu_3, 2\nu_3-1, 0) & \text{for } \frac{\nu_2+1}{2} \leq \nu_3 \leq \frac{2}{3}, \\ (1-\nu_3, 1-\nu_3, 1-\nu_3, 3\nu_3-2) & \text{for } \nu_3 \geq \max\left\{\frac{\nu_2+1}{2}, \frac{2}{3}\right\}. \end{array}$$

The last player's power also increases in 3-player games with an empty core. From the proof of Proposition 1, part c2), we know that

$$s^{\alpha}(v) = (v(N) - v(\{2,3\}), v(N) - v(\{1,3\}), v(\{1,3\}) + v(\{2,3\}) - v(N)).$$

This means that only 2-player coalitions including player 3, that is coalitions of the form $\{i, 3\}$, satisfy $s_i^{\alpha}(v) + s_3^{\alpha}(v) = v(\{i,3\})$, while $s_1^{\alpha}(v) + s_2^{\alpha}(v) < v(\{1,2\})$, i.e. players 1 and 2 have no chance of doing without player 3.

4 The main result

We define here the bargaining mechanism. First, a random order is chosen. As in the previous section, we assume that this order is $\alpha = (12...n)$. The mechanism has n stages. In the first stage, player 1 makes a demand $d_1 \in \mathbb{R}$. In the second stage, player 2, aware of player 1's choice, makes a demand $d_2 \in \mathbb{R}$, and so on. When player n's turn comes, he faces a vector $(d_i)_{i \in P_n^{\alpha}} \in \mathbb{R}^{P_n^{\alpha}}$ of demands. He then selects a group of players to exclude, denoted $E \subset P_n^{\alpha}$.

This means that the coalition $N \backslash E$ forms, and player n gets its resources by paying d_i to every player $i \in P_n^\alpha \backslash E$. The final payoff is d_i for every player $i \in P_n^\alpha \backslash E$, $v(N \backslash E) - \sum_{i \in P_n^\alpha \backslash E} d_i$ for player n, and $v(\{i\})$ for every $i \in E$. We say then that players in E are *excluded*.

The next theorem is our main result, and it shows that the selective value arises in the bargaining mechanism as the only expected subgame perfect equilibrium payoff.

Theorem 1 *For strictly zero-monotonic games, there exists a unique expected subgame perfect equilibrium payoff, and it is the selective value.*

To prove this result, we need the following lemma. We denote by $M(\alpha, i, d)$ the subgame which begins when players in P_i^α have stated their demands $(d_j)_{j \in P_i^\alpha} \in \mathbb{R}^{P_i^\alpha}$ and it is player i's turn. If $i = 1$, we write $M(\pi, 1)$.

Lemma 3 *Let v be a strictly zero-monotonic game. Assume we are in a subgame perfect equilibrium of the subgame $M(\pi, i, d)$ and $\pi(i) < n$ (i.e. player i has to make a demand). Then, player i demands $d_i = v_d(N \backslash P_i^\pi) - v_d\left(N \backslash \overline{P_i^\pi}\right)$, and he is not excluded.*

Proof. We proceed by backwards induction. Assume we are in the last round, i.e. we are in the subgame $M(\alpha, n, d)$ for some $d \in \mathbb{R}^{P_n^\alpha}$. Since we are in equilibrium, player n should exclude a coalition $E \subset P_n^\alpha$ such that

$$E \in \arg\max_{T \subset P_n^\alpha} \left\{ v(N \backslash T) - \sum_{j \in P_n^\alpha \backslash T} d_j \right\}. \quad (6)$$

Assume now that it is the turn of player $i < n$, i.e. we are in the subgame $M(\alpha, i, d)$ for some $(d_j)_{j \in P_i^\alpha} \in \mathbb{R}^{P_i^\alpha}$.

Claim: Player $i < n$ is excluded if and only if $d_i > v_d(N \backslash P_i^\alpha) - v_d\left(N \backslash \overline{P_i^\alpha}\right)$.

The implication of this Claim is as follows: Since player i obtains $v(\{i\})$ if excluded, by Lemma 2 it is optimal for him not to be excluded. Thus, $d_i \leq v_d(N \backslash P_i^\alpha) - v_d\left(N \backslash \overline{P_i^\alpha}\right)$. Clearly, no $d_i < v_d(N \backslash P_i^\alpha) - v_d\left(N \backslash \overline{P_i^\alpha}\right)$ can be part of an equilibrium because $d_i + \varepsilon$ is a better reservation price for i without being excluded. Thus, $d_i = v_d(N \backslash P_i^\alpha) - v_d\left(N \backslash \overline{P_i^\alpha}\right)$ and player i is not excluded.

We now prove the Claim. Consider the subgame $M(\alpha, i, d)$ for some $(d_j)_{j \in P_i^\alpha} \in \mathbb{R}^{P_i^\alpha}$. Assume first that $d_i < v_d(N \backslash P_i^\pi) - v_d\left(N \backslash \overline{P_i^\pi}\right)$. We prove that player i is not excluded, i.e. $i \notin E$ with E satisfying (6). Suppose, on the contrary, that $i \in E$. We show that d_i is low enough that there is $E' \subset P_n^\alpha$ such that player n strictly prefers to exclude E'. If $i = n-1$, it is clear that no player following i is excluded. If $i < n-1$, we can assume the same thing by the induction hypothesis. Thus, $E \subset \overline{P_i^\alpha}$.

Let

$$E' \in \arg\max_{T \subset P_i^\alpha} \left\{ v(N \backslash T) - \sum_{j \in P_n^\alpha \backslash T} d_j \right\}.$$

Then

$$\begin{aligned} v(N \backslash E') - \sum_{j \in P_n^\alpha \backslash E'} d_j &= \max_{T \subset P_i^\alpha} \left\{ v(N \backslash T) - \sum_{j \in P_n^\alpha \backslash T} d_j \right\} \\ &= v_d(N \backslash P_i^\alpha) - \sum_{j \in P_n^\alpha \backslash P_i^\alpha} d_j \\ &> v_d\left(N \backslash \overline{P_i^\alpha}\right) - \sum_{j \in (P_n^\alpha \backslash \{i\}) \backslash P_i^\alpha} d_j \\ &= \max_{T \subset \overline{P_i^\alpha} : i \in T} \left\{ v(N \backslash T) - \sum_{j \in P_n^\alpha \backslash T} d_j \right\} \\ &\geq v(N \backslash E) - \sum_{j \in P_n^\alpha \backslash E} d_j. \end{aligned}$$

But this contradicts (6). Thus, $i \notin E$ and player i's final payoff is d_i.

Assume now $d_i > v_d\left(N\backslash P_i^{\alpha}\right) - v_d\left(N\backslash\overline{P_i^{\alpha}}\right)$. We have to prove that player i is excluded, namely $i \in E$. Suppose, on the contrary, that $i \notin E$. Again, we show that d_i is long enough that there is $E' \subset P_n^{\alpha}$ such that player n strictly prefers to exclude E'. If $i = n-1$, it is clear that no player following i is excluded. If $i < n-1$, we can assume the same thing by the induction hypothesis. Thus, $E \subset P_i^{\alpha}$. Let

$$E' \in \underset{T\subset\overline{P_i^{\alpha}}:i\in T}{\arg\max}\left\{v\left(N\backslash T\right) - \sum_{j\in P_n^{\alpha}\backslash T} d_j\right\}.$$

Then

$$\begin{aligned}
v\left(N\backslash E'\right) - \sum_{j\in P_n^{\alpha}\backslash E'} d_j &= \max_{T\subset\overline{P_i^{\alpha}}:i\in T}\left\{v\left(N\backslash T\right) - \sum_{j\in P_n^{\alpha}\backslash T} d_j\right\} \\
&= \max_{T\subset P_i^{\alpha}}\left\{v\left(N\backslash\left(T\cup\{i\}\right)\right) - \sum_{j\in P_n^{\alpha}\backslash(T\cup\{i\})} d_j\right\} \\
&= v_d\left(N\backslash\overline{P_i^{\alpha}}\right) - \sum_{j\in P_n^{\alpha}\backslash\overline{P_i^{\alpha}}} d_j \\
&> v_d\left(N\backslash P_i^{\alpha}\right) - \sum_{j\in P_n^{\alpha}\backslash P_i^{\alpha}} d_j \\
&= \max_{T\subset P_i^{\alpha}}\left\{v\left(N\backslash T\right) - \sum_{j\in P_n^{\alpha}\backslash T} d_j\right\} \\
&\geq v\left(N\backslash E\right) - \sum_{j\in P_n^{\alpha}\backslash E} d_j.
\end{aligned}$$

Again, this contradicts (6). Thus, player i is excluded and his final payoff is $v\left(\{i\}\right)$.

Since it is optimal for player i not to be excluded, he commits to exactly $v_d\left(N\backslash P_i^{\alpha}\right) - v_d\left(N\backslash\overline{P_i^{\alpha}}\right)$. ■

We now proceed to prove the main theorem.

Proof of Theorem 1. By Lemma 3, the only possible payoff in equilibrium is $s^{\alpha}(v)$. We must prove that there exists an equilibrium supporting $s^{\alpha}(v)$. We consider the following strategies: In the subgame $M(\alpha, i, d)$, player i demands $d_i = v_d(N \backslash P_i^{\alpha}) - v_d\left(N \backslash \overline{P_i^{\alpha}}\right)$. In the subgame $M(\alpha, n, d)$, player n excludes a coalition $E \subset P_n^{\alpha}$ such that

$$E \in \arg\max_{T \subset P_n^{\alpha}} \left\{ v(N \backslash T) - \sum_{j \in P_n^{\alpha} \backslash T} d_j \right\}. \tag{7}$$

If there are more than one maximizer, player n chooses a coalition E with minimum cardinality.

By the Claim in the proof of Lemma 3, it is clear that these strategies constitute a subgame perfect equilibrium. Moreover, the final payoff is $s^{\alpha}(v)$. Since the order is randomly chosen, we deduce that the only final expected payoff is the selective value. ■

5 Concluding remarks

An immediate consequence of Lemma 3 is that the equilibrium payoff of any player depends on the identity of the players who come before and after him, but not on the way they are ordered. Hence, they do not need to know the order in advance. In fact, the bargaining mechanism could have been defined by choosing the first player at random; after this player makes his demand, another player is chosen at random, and so on.

If we consider (non-strictly) zero-monotonic games, there exists a subgame perfect equilibrium whose expected payoff outcome coincides with the selective value. A possible equilibrium is the one presented in the proof of Theorem 1, where the group of minimal cardinality is excluded in case of indifference between several groups E. However, there may exist subgame perfect equilibria whose associated expected final payoff outcome is not the selective value. Consider the next example.

Example 4 *Let* $n = 3$ *and* v *be given by* $v(N) = v(\{1,3\}) = v(\{2,3\}) = 1$ *and* $v(S) = 0$ *otherwise. The selective value of* v *is the only core allocation* $(0,0,1)$*. We consider the following strategies: Players* 2 *and* 3 *play according to the strategies described in the proof of Theorem 1, which implement the selective value. However, if the set given in (7) contains more than one coalition, player* 3 *will exclude the first coalition in (7) given the preference relation* $\emptyset \succ \{1\} \succ \{2\} \succ \{1,2\}$*. Moreover, player* 1 *plays according to these strategies except when he is first in the order, where he demands* $d_1 = 1$*.*

It is not difficult to check that these strategies constitute a subgame perfect equilibrium. When the order is different from (123)*, the final payoff is* $(0,0,1)$*. When the order is* (123)*, the final payoff is* $(0,1,0)$*. Hence, the selective value is not achieved.*

If we want to obtain the selective value for general zero-monotonic games, we have to make additional assumptions. For example, let us consider the following tie-breaking rule:

- If a player i is indifferent to demanding d'_i or d_i and $d'_i < d_i$, he strictly prefers to demand d'_i.
- If the last player is indifferent to excluding E' or E and $E' \subsetneq E$, he strictly prefers to exclude E'.

With this tie-breaking rule, we can prove that Theorem 1 may be extended to any zero-monotonic game.

Vidal-Puga and Bergantiños (2003) model this tie-breaking rule by "punishing" with a small penalty $\varepsilon > 0$ the players involved in an exclusion. We can do the same thing in our model. In particular, let us assume that each excluded player must pay $\varepsilon > 0$. We call this modified mechanism the *ε-bargaining mechanism.* Its structure is otherwise the same as before. The only difference lies in the payoff function. When the last player presents a coalition E of excluded players, the final payoff is $v(\{i\}) - \varepsilon$ for each $i \in E$.

By similar arguments to those of Theorem 1, we can prove that, for any $\varepsilon > 0$, the ε-bargaining mechanism yields the selective value as final expected outcome for any zero-monotonic game.

Remark 3 *The selective value is also the unique expected subgame perfect equilibrium payoff if the penalty to the excluded players is agent-dependent, i.e. any player i has a penalty $\varepsilon(i) > 0$ for being excluded.*

6 Appendix

Proof of Proposition 1. **a**) We prove that, in convex games, the balanced contributions and the selective balanced contributions coincide; i.e. $s^\alpha(v) = m^\alpha(v)$. This means that the marginal contribution of a player $i \in N$ in the room does not change if players outside the room are not available, that is,

$$v\left(N \backslash \overline{P_i^\alpha}\right) \geq v\left(N \backslash (T \cup \{i\})\right) - \sum_{j \in P_i^\alpha \backslash T} s_j^\alpha(v) \tag{8}$$

for all $T \subset P_i^\alpha$ and

$$s_i^\alpha(v) = m_i^\alpha(v). \tag{9}$$

We proceed by induction on i. For $i = 1$, (8) is trivial and (9) coincides with (4). Let $i > 1$ and assume that (8)-(9) hold for $1, 2, ..., i-1$. Let $T \subset P_i^\alpha$. Then, (8) is equivalent to

$$v\left(N \backslash \overline{P_i^\alpha}\right) \geq v\left(N \backslash (T \cup \{i\})\right) - \sum_{j \in P_i^\alpha \backslash T} m_j^\alpha(v). \tag{10}$$

We prove (10) by inverse induction on $|T|$. For $T = P_i^\alpha$, (10) holds trivially. Assume (10) holds for all coalitions $S \subset P_i^\alpha$ such that $|T| < |S| \leq |P_i^\alpha|$. Let i^* be the first player in $P_i^\alpha \backslash T$, i.e. the player with the lowest index in $P_i^\alpha \backslash T$. This means that all his predecessors (if any) belong to T, i.e. $P_{i^*}^\alpha \subset T$.

Let $T^* := T \cup \{i^*\} \subset P_i^\alpha$. By the induction hypothesis

$$\begin{aligned} v\left(N \backslash \overline{P_i^\alpha}\right) &\geq v\left(N \backslash \left(T^* \cup \{i\}\right)\right) - \sum_{j \in P_i^\alpha \backslash T^*} m_j^\alpha (v) \\ &= v\left(N \backslash \left(T \cup \{i, i^*\}\right)\right) + m_{i^*}^\alpha (v) - \sum_{j \in P_i^\alpha \backslash T} m_j^\alpha (v) . \end{aligned} \tag{11}$$

But $\overline{P_{i^*}^\alpha} \subset T \cup \{i, i^*\}$. Thus, $N \backslash \overline{P_{i^*}^\alpha} \supset N \backslash \left(T \cup \{i, i^*\}\right)$, and by convexity,

$$v\left(N \backslash P_{i^*}^\alpha\right) - v\left(N \backslash \overline{P_{i^*}^\alpha}\right) \geq v\left(N \backslash \left(T \cup \{i\}\right)\right) - v\left(N \backslash \left(T \cup \{i, i^*\}\right)\right). \tag{12}$$

Since $m_{i^*}^\alpha (v) = v\left(N \backslash P_{i^*}^\alpha\right) - v\left(N \backslash \overline{P_{i^*}^\alpha}\right)$, we apply (11) and (12) to obtain (10).

We now prove (9)

$$\begin{aligned} s_i^\alpha (v) &= v(N) - \sum_{j \in P_i^\alpha} s_j^\alpha (v) - \max_{T \subset P_i^\alpha} \left\{ v\left(N \backslash \left(T \cup \{i\}\right)\right) - \sum_{j \in P_i^\alpha \backslash T} s_j^\alpha (v) \right\} \\ &= v(N) - \sum_{j \in P_i^\alpha} \left[v\left(N \backslash P_j^\alpha\right) - v\left(N \backslash \overline{P_j^\alpha}\right) \right] - v\left(N \backslash \overline{P_i^\alpha}\right) \\ &= v(N) - \left[v(N) - v\left(N \backslash P_i^\alpha\right) \right] - v\left(N \backslash \overline{P_i^\alpha}\right) \\ &= v\left(N \backslash P_i^\alpha\right) - v\left(N \backslash \overline{P_i^\alpha}\right) = m_i^\alpha (v) . \end{aligned}$$

This completes the proof of part a).

b1) Assume first that $T = \emptyset$. Since no player is necessary to form a winning coalition, the marginal contribution of the first player (i.e. his demand) is zero. Thus, the situation inside the room does not change (i.e. again, no player is necessary to form a winning coalition). This means that all the players leave the room demanding zero, and the last player obtains 1 for getting the resources of the other players for free. Since each player has the same probability of being last, the expected final payoff is the same for all players.

b2) Assume now that $T \neq \emptyset$. Then, the same reasoning as before applies for non veto players. However, the first veto player has a marginal contribution of 1, because no winning coalition can be formed without him. Thus, the first veto player in the order obtains 1 and the remaining players 0. Since each veto player has the same probability of being first, the expected final payoff is the same for all veto players.

c1) It is straightforward to show that[4], under our hypothesis,

$$\max\{y_i : y \in C(v)\} = v(N) - v(N\backslash i) \tag{13}$$

for all $i \in N$.

We define x^α as the vertex of $C(v)$ associated to α. Namely

$$\begin{aligned} x_1^\alpha &= \max\{y_1 : y \in C(v)\} \\ x_2^\alpha &= \max\{y_2 : y \in C(v), y_1 = x_1\} \\ x_3^\alpha &= v(N) - x_1^\alpha - x_2^\alpha. \end{aligned}$$

We prove that $x^\alpha = s^\alpha(v)$. By (13)

$$x_1^\alpha = v(N) - v(23) = s_1^\alpha(v).$$

Moreover

$$\begin{aligned} s_2^\alpha(v) &= v(N) - \max\{v(13), v(3) + s_1^\alpha(v)\} \\ &= v(N) - \max\{v(13), v(N) - v(23) + v(3)\}. \end{aligned}$$

It is enough to prove that $s_2^\alpha(v) = x_2^\alpha$. There are two cases:

[4] In this section, we use $v(N\backslash i)$ instead of the more cumbersome $v(N\backslash\{i\})$. Similarly, $v(ij) = v(\{i,j\})$ and so on.

Case 1: $v(13) \geq v(N) - v(23) + v(3)$. Then $s_2^{\alpha}(v) = v(N) - v(13)$. We show that

$$v(N) - v(13) = \max\{y_2 : y \in C(v), y_1 = v(N) - v(23)\} = x_2^{\alpha}.$$

Let $y \in C(v)$ be such that $y_1 = v(N) - v(23)$. Then

$$y_2 = v(N) - y_1 - y_3 = v(23) - y_3.$$

Since $y \in C(v)$, we have $y_1 + y_3 \geq v(13)$ and thus

$$y_2 \leq v(23) + y_1 - v(13) = v(N) - v(13).$$

Therefore, $x_2^{\alpha} \leq v(N) - v(13)$.

Let $y \in C(v)$ such that $y_1 = v(N) - v(23)$ and $y_2 < v(N) - v(13)$. Let $y^{\varepsilon} := y + (0, \varepsilon, -\varepsilon)$ with $0 < \varepsilon < v(N) - v(13) - y_2$. So, $y_1^{\varepsilon} = v(N) - v(23)$. It is straightforward to show that $y^{\varepsilon} \in C(v)$.

Thus, $x_2^{\alpha} = v(N) - v(13) = s_2^{\alpha}(v)$.

Case 2: $v(13) < v(N) - v(23) + v(3)$. Then $s_2^{\alpha}(v) = v(23) - v(3)$. We show that

$$v(23) - v(3) = \max\{y_2 : y \in C(v), y_1 = v(N) - v(23)\} = x_2^{\alpha}.$$

Let $y \in C(v)$ be such that $y_1 = v(N) - v(23)$. Then

$$y_2 = v(N) - y_1 - y_3 = v(23) - y_3.$$

Since $y \in C(v)$, we have $y_3 \geq v(3)$ and thus

$$y_2 \leq v(23) - v(3).$$

Therefore, $x_2^{\alpha} \leq v(N) - v(3)$.

Let $y \in C(v)$ be such that $y_1 = v(N) - v(23)$ and $y_2 < v(23) - v(3)$. Let $y^{\varepsilon} := y + (0, \varepsilon, -\varepsilon)$ with $0 < \varepsilon < v(23) - v(3) - y_2$. So, $y_1^{\varepsilon} = v(N) - v(23)$. It is straightforward to show that $y^{\varepsilon} \in C(v)$.

Thus, we conclude that $x_2^\alpha = v(23) - v(3) = s_2^\alpha(v)$.

c2) For every vector $x \in \mathbb{R}^N$, and every $S \subset N$, we define the *excess* of S at x as the number $e_S(x) := v(S) - \sum_{i\in S} x_i$. We use the next lemma, due to Kohlberg (1971):

Lemma 4 *Let $\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, ...$ be the sets of coalitions of highest excess at x, second highest, third highest, etc. Let $\mathcal{D}_t = \mathcal{B}_1 \cup \mathcal{B}_2 \cup ... \cup \mathcal{B}_t$. Then, x is the prenucleolus iff each $\mathcal{D}_t$ is a balanced collection*[5]*.*

First, we compute $s^\alpha(v)$. By (4), $s_1^\alpha(v) = v(N) - v(23)$. To compute $s_2^\alpha(v)$, we note that the payoff

$$x := (v(N) - v(23), v(23) - v(3), v(3))$$

satisfies $\sum_{i\in S} x_i \geq v(S)$ for all $S \neq \{1,3\}$ (by zero-monotonicity). Since the core is empty, $x_1 + x_3 < v(13)$. Hence,

$$\begin{aligned} s_2^\alpha(v) &= v(N) - \max\{v(13), v(3) + s_1^\alpha(v)\} \\ &= v(N) - \max\{v(13), x_3 + x_1\} = v(N) - v(13). \end{aligned}$$

Since $s^\alpha(v)$ is efficient, $s_3^\alpha(v) = v(13) + v(23) - v(N)$. Performing the required calculations, we obtain the selective value for player 1 as

$$\sigma_1(v) = \frac{1}{3}[v(N) + v(12) + v(13) - 2v(23)].$$

The expressions for $\sigma_2(v)$ and $\sigma_3(v)$ are analogous. We now compute the excesses at $\sigma(v)$.

The excess of $\{1,2\}$ at $\sigma(v)$ is

$$e_{12}(\sigma(v)) = \frac{1}{3}[v(12) + v(13) + v(23) - 2v(N)].$$

[5] A collection $\mathcal{C} = \{S_1, S_2, ..., S_t\}$ of coalitions is *balanced* if there exists a vector $\lambda \in \mathbb{R}^{\mathcal{C}}$ with $\lambda_S > 0$ for all $S \in \mathcal{C}$ such that $\sum_{S\in\mathcal{C}: i\in S} \lambda_S = 1$ for all $i \in N$.

By symmetry, $e_{12}(\sigma(v)) = e_{13}(\sigma(v)) = e_{23}(\sigma(v))$. It is well-known that a zero-monotonic 3-person game v has an empty core if and only if

$$v(12) + v(13) + v(23) > 2v(N)$$

and thus the above excesses are positive.

Since the excess of $\{i\}$ at $\sigma(v)$ is clearly nonpositive for any $i \in N$, it follows from Lemma 4 that $\sigma(v)$ is the prenucleolus of v. ■

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