Comments on: Games with a Permission Structure*

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Van den Brink (2017) presents an excellent survey on how a permission structure affects the sharing of benefits from cooperation among a finite set of players. The survey covers from the simplest and most intuitive permission structure, given by a directed graph, to a more general one, given by the abstract concept of *normal antimatroid* (Dilworth, 1940). This generalization allows to include very particular cases such as the one presented in Example 2, where permission is hold not by a fixed set of players but as the result of a voting game. I believe normal antimatroids could be the most general concept that maintains the essential ideas of a permission structure.

On the other hand, and despite the fact that other solution concepts are briefly commented in the concluding remarks section, the results are focused on the Shapley value. The Shapley value is already considered as (one of) the most relevant solution concept(s) in transferable utility (TU) games. Yet, I believe it is specially relevant in the case where cooperation is influenced by a graph (or, in general, under the influence of an antimatroid). This is so for two main reasons.

Firstly, as already pointed out in the seminal paper by Myerson (1977), there exists a natural property of fairness that characterizes the Shapley value of the game that results from restricting cooperation to coalitions that are connected under a graph structure. This fairness property simply states that adding (or deleting) a link would affect symmetrically both adjacent nodes. Yet the property is strong enough to single out a unique solution, the Shapley value of the modified game.

The present survey shows how this natural result can be extended to preference structures which, as opposed to Myerson's approach, not only determine the feasible coalitions,

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but also a dependence relation among the players. Yet, this dependence relation may have two very different interpretations, depending on whether it allows new possibilities (this is the *disjunctive* approach, where a player needs permission from any predecessor in the preference relation) or new restrictions (this is the *conjunctive* approach, where a player need permission from all predecessors in the preference relation).

Of course, as it is clearly explained by the author, the correct interpretation (disjunctive or conjunctive) depends on the application where the model is due to be applied. The different axiomatizations provide a clear insight into this dichotomy. Properties such as that "any player should receive at least as much as her immediate subordinates" makes much sense in the conjunctive approach (when the immediate subordinates are useless without her participation), and it characterizes the conjunctive Shapley value with other basic properties.

It is also worth mentioning that all the properties given in the axiomatizations are independent, providing a tight characterization of the results.

Most of the properties presented in the review have their natural counterpart in the class of general TU games. Of course, they mainly refer to the permission structure.

As already pointed out by Young (1985), the Shapley value satisfies nice monotonic properties. Roughly speaking, monotonicity says that a (weak) increase in the capabilities (in production or in permission) of a player should turn into a (weak) increase in her final payoff. Hence, monotonicity is a fundamental property in order to ensure that the players have incentives to increase their own capabilities.

It is remarkable that this idea of monotonicity, applied to the permission structure, appears in several important characterization results, such as Theorem 1, Theorem 2, Theorem 10, Theorem 11, Theorem 19, Theorem 21 and Theorem 24, under the generic name of (local) structural monotonicity.

As a counterpart of monotonicity, Young (1985) also showed that in general it is incompatible with core selection, in the sense that a monotonic solution cannot belong to the core of some balanced games. This may be an important drawback of the Shapley value.

This leads to the second reason I believe the Shapley value is especially relevant for games with a permission structure. Note that the permission structure creates a particular subclass of TU games where the Shapley value may be a core selector. For example, Graham et al. (1990) prove that this is the case in auction games (Section 6.1). In general, the conjunctive approach, where the preference structure restricts the possibilities of cooperation, is quite promising in this aspect. Furthermore, as pointed out by the author, the core is nonempty in some relevant situations, for example when the initial TU game is superaditive and the communication graph has no cycles. So, finding under which circumstances the Shapley value belongs to the core may be a promising field for future research.

Moreover, even if the Shapley value lies outside the core, this may not be a relevant issue. In fact, games with a permission structure may be examples of situations where it is not. Of course, this would depend on the application we want to model, but the example of hierarchical structure firms (Section 6.2) is a paradigmatic case where the workers have no direct access to the production technology and hence core selection is a minor issue.

In conclusion, the survey covers most, if not all, of the relevant permission structures that may influence a TU game with a finite number of players. Furthermore, it focuses on the Shapley value, which is a very relevant solution concept in TU games in general and TU games with permission structure in particular, using very natural and relevant properties.

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